

Social Learning and the Design of New Experience Goods

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Consumers often consult the reviews of their peers before deciding whether to purchase a new experience good; however, their initial quality expectations are typically set by the product's observable attributes. This paper focuses on the implications of social learning for a monopolist firm's choice of product design. In our model, the firm's design choice determines the product's ex ante expected quality, and designs associated with (stochastically) higher quality incur higher costs of production. Consumers are forward-looking social learners, and may choose to strategically delay their purchase in anticipation of product reviews. In this setting, we find that the firm's optimal policy differs significantly depending on the level of the ex ante quality uncertainty surrounding the product. In comparison to the case where there is no social learning, we show that (i) when the uncertainty is relatively low, the firm opts for a product of inferior design accompanied by a lower price, while (ii) when the uncertainty is high, the firm chooses a product of superior design accompanied by a higher price; interestingly, we find that the product's expected quality decreases either in the absolute sense (in the former case), or relative to the product's price (in the latter case). We further establish that, contrary to conventional knowledge, social learning can have an ex ante negative impact on the firm's profit, in particular when the consumers are sufficiently forward-looking. Conversely, we find that the presence of social learning tends to be beneficial for the consumers only provided they are sufficiently forward-looking.

Key words: Bayesian social learning, endogenous product quality, product design, strategic consumer behavior, applied game theory

1. Introduction

The influence of consumer reviews on the purchase decisions of potential product buyers has grown dramatically in the last decade, with recent surveys suggesting that up to 69% of consumers now consult peer reviews before deciding whether to purchase a product (e.g., Hinckley 2015, Mintel.com 2015). This figure can only be expected to increase in the future: on one hand, rapid innovation

and technological advancements are rendering more and more products increasingly complex and difficult to evaluate before purchase (e.g., consumer electronics such as smartphones, media items such as movies, digital goods such as software, etc.); on the other, the proliferation of online forums and platforms hosting product reviews (e.g., *Amazon*, *TripAdvisor*, *Yelp*, etc.) is providing consumers with unprecedented ease-of-access to the post-purchase opinions of their peers.

From the firm’s perspective, these trends translate into increasing pressure to understand how various product policies interact with review-based *social learning* (SL) and to optimize these policies accordingly (e.g., McKinsey 2010). In working towards such an understanding, a recent stream of literature has adapted the classic theory of “experience goods” (Nelson 1970) to allow for the exchange of post-purchase consumer opinions through product reviews.¹ So far, efforts in this area of research have focused primarily on developing insights regarding the optimal pricing of experience goods, implicitly assuming that the firm is endowed with a product whose attributes are specified exogenously (e.g., Crapis et al. 2015, Papanastasiou and Savva 2016, Yu et al. 2015). This paper takes the natural next step of recognizing the firm’s role in choosing these attributes – a decision we refer to as “product design” – and investigating how this decision interacts with the process of review-based SL.

We focus, in particular, on product-design choices pertaining to ex ante observable “quality attributes” (i.e., attributes which are valued by all consumers in a “more-is-better” fashion); for example, in the design of a new smartphone, these attributes may include the processor speed, screen definition, and memory capacity. Thus, product design in the context of this paper exhibits the familiar tradeoff between adding or enhancing product attributes that increase the product’s perceived quality (so as to increase the consumers’ willingness-to-pay), and avoiding higher costs of production (so as to maintain higher profit margins). Although this tradeoff has been previously studied from numerous perspectives (e.g., Villas-Boas 1998, Netessine and Taylor 2007, Jerath et al. 2015), existing work has by-and-large treated products as “search goods,” in that, conditional on the product design, there is no quality uncertainty or, equivalently, no opportunity for consumers to interact to resolve such uncertainty.² As a result, the implications of SL for product design remain as-of-yet unclear.

Taking the above into account, in this paper we consider two main research questions. First, how should a firm incorporate review-based SL into its choice of product design and, by extension, how does SL affect the quality of new experience goods? Second, how does the interaction between

¹ The term “experience good” refers to a product whose quality is difficult to assess before purchase.

² In reality, experience goods can exhibit significant quality uncertainty even when their design is perfectly observable (e.g., owing to the complex interaction between a product’s components); recent notable examples include Apple’s “bending” iPhone 6 (Forbes 2015) and Samsung’s “exploding” Galaxy Note 7 (Cnet.com 2016).

SL and product design impact the firm’s profit and the consumers’ surplus? To investigate these questions, we develop a two-period model that captures the interactions between a monopolistic firm and a population of strategic (i.e., forward-looking) rational consumers. At the beginning of the selling season, the firm chooses the product’s price and design. In our model, the latter determines the product’s quality only up to the expected value, and designs associated with higher expected quality incur higher per unit production costs. Consumers are Bayesian social learners, and those who choose to defer their purchase decision to the second period can benefit by observing the reviews of first-period buyers, thereby reducing their uncertainty over product quality. Our equilibrium analysis of this model yields insights along three dimensions, which can be summarized as follows:

- (i) **Product Design.** We show that the firm’s policy choice in the presence of SL differs significantly depending on the magnitude of the quality uncertainty surrounding the product. In particular, when the uncertainty is relatively low, the firm focuses on increasing early adoption through a policy that involves an inferior product design and a lower price (as compared to the case where there is no SL); by contrast, when the uncertainty is high, the firm focuses instead on increasing its profit margin, through a policy that involves a superior design and a higher price. Interestingly, we observe that the product’s expected quality in the presence of SL is lower either in the absolute sense, or relative to the product’s price. At the same time, we find that when the firm chooses a product of lower expected quality, the accompanying price often results in a product of higher relative quality; this is in contrast to the case where the firm adjusts either price or design unilaterally, where the optimal policy is always one of lower relative quality.
- (ii) **Firm Profit.** We establish that SL can have a net negative impact on the firm’s ex ante profit, in particular when the consumers are highly strategic. As the fraction of buyers who write reviews increases (thus increasing the pervasiveness of SL among consumers), we find that the consumers’ increased tendency to delay purchase in anticipation of reviews may in fact *reverse* the benefits of increased consumer learning for the firm. Thus, depending on the level of strategic consumer behavior, we observe that the firm’s ex ante profit may be maximized when all (low degree), some (intermediate degree), or none (high degree) of the consumers engage in writing reviews. This result offers a new perspective on the interaction between SL and strategic consumer behavior, suggesting that this can be more detrimental for the firm than previously thought (e.g., Papanastasiou and Savva 2016, Yu et al. 2015).
- (iii) **Consumer Surplus.** Although the exchange of information through product reviews generates value for consumers by allowing for better-informed purchase decisions, we find that

SL in most cases has a negative (from the consumers' perspective) impact on the firm's chosen price-and-design combination. As a result, we show that the presence of SL tends to be detrimental for the consumers' ex ante surplus, unless they are sufficiently forward-looking in their purchase decisions; interestingly, when this is not the case, we further observe that consumers are worse off when more of their peers engage in writing reviews.

2. Related Literature

The literature on SL has its origins in economics and the seminal papers by Banerjee (1992) and Bikhchandani et al. (1992), which illustrate how agents endowed with private information on some unobservable state of the world can learn from each other through observation of each others' actions. While earlier work focused on learning outcomes in homogeneous societies (i.e., agents with homogeneous preferences) and simple social structures (i.e., sequential actions and observations), more recent work has expanded the scope of study to heterogeneous societies and more complicated social networks (e.g., Acemoglu et al. 2011, Jadbabaie et al. 2012, Lobel and Sadler 2015, Manshadi and Misra 2016, Papanastasiou 2017, Zhang et al. 2017).

With the proliferation of online opinion forums and review platforms where consumers exchange information on products and services, SL has recently received significant attention in the fields of Operations Management and Marketing: Chen and Xie (2010) identify online reviews as a new element in the marketing communications mix, while Debo and Veeraraghavan (2009) highlight the importance of consumers-to-consumer learning in shaping firms' operational strategies.³ Several recent papers build on the latter notion, studying operational decisions in environments where consumers' purchase decisions are influenced by the actions and opinions of their peers. Papanastasiou et al. (2016) illustrate how a review platform can employ information obfuscation to promote exploration in a multi-product setting; Hu et al. (2015) analyze inventory decisions when the consumers' choices between two substitutable products are affected by the choice of their predecessors, while Debo and Van Ryzin (2009) and Papanastasiou et al. (2015) explore the use of strategic stockouts as a means to increase demand through SL; Momot et al. (2016) explore the value of social-network information when selling to conspicuous consumers; Crapis et al. (2015) point out the benefits of pricing policies that take the presence of SL into account. Closest in spirit to our work are two recent papers by Papanastasiou and Savva (2016) and Yu et al. (2015) that consider optimal pricing policies in the presence of review-based SL and strategic consumer behavior.⁴ In these papers,

³For empirical work that highlights the connection between consumer reviews and product sales, see Archak et al. (2011), Dellarocas et al. (2007), Chevalier and Mayzlin (2006), and references therein.

⁴For operational implications of strategic consumer behavior in other contexts, see Su (2007), Swinney (2011), Cachon and Feldman (2015), and references therein.

the firm sells a product whose quality attributes are exogenously specified; by contrast, our paper focuses on the implications of the firm’s costly choice of such attributes.

The firm’s choice of quality attributes is a central decision in another stream of research, which is primarily concerned with “search goods.” In single-product settings, Xu (2009) highlights the implications of the distribution-channel structure on a firm’s quality and pricing decisions, while Shi et al. (2013) focus on the impact of different types of consumer heterogeneity (i.e., vertical and horizontal). Jerath et al. (2015) consider how quality decisions are affected by demand uncertainty and inventory risk. In multi-product settings, the literature on product-line design focuses on how multiple products of different qualities may be offered in order to segment consumers based on their heterogeneity in willingness-to-pay for quality (e.g., Mussa and Rosen 1978, Moorthy 1984). More recent work in this area considers product-line decisions in the context of competition (Desai 2001), development-intensive products (Krishnan and Zhu 2006), different distribution-channel structures (Villas-Boas 1998), and different production technologies (Chen et al. 2013, Netessine and Taylor 2007), among others. A recent paper by Godes (2016) studies quality choice in a setting where persuasive word-of-mouth communication between consumers occurs exogenously to the firm’s policy, and argues that product quality should increase in the intensity of such communications. Among other differences with this work, a central feature of our model is the endogenous nature of word-of-mouth communications, both in terms of their intensity and in terms of their content; once this endogeneity is taken into account, we show that optimal product quality often decreases in the presence of SL. Significantly, in all of the aforementioned papers, product quality is perfectly known conditional on the firm’s design choice (i.e., in this work, design and quality are synonymous). By contrast, in this paper we focus on the design of “experience goods,” which are characterized by ex ante quality uncertainty despite their design being perfectly observable.

3. Model Description

We consider a monopolist firm selling a new experience good over a single selling season. The product’s quality, \hat{q} , is the sum of two components, q and ϵ . Component q is observable and represents the *product design*, which is chosen by the firm; by contrast, component ϵ is an unobservable quality shock, which is drawn by nature from the Normal distribution $N(0, \sigma_\epsilon^2)$. The product design q is a summary measure of all observable product attributes which consumers value in a “more-is-better” manner (e.g., for a new smartphone, these may include the processor speed, screen definition, memory capacity, etc.). Since in our model the firm’s design choice determines the product’s ex ante expected quality, we occasionally refer to q directly as the firm’s “quality choice.” Consistent with the literature on endogenous product quality, we assume that a product of expected quality q

incurs a quadratic per unit production cost $c(q) = q^2$ (e.g., Desai 2001, Netessine and Taylor 2007).⁵ On the other hand, nature’s shock ϵ captures the ex ante uncertainty associated with the product’s quality, which may be attributed to product characteristics which are ex ante unobservable, such as the product’s usability, durability and usefulness, and/or to how the product’s various components interact with each other (see also Papanastasiou and Savva 2016, Yu et al. 2015).

The market consists of a continuum of consumers of total mass normalized to one, and sales of the product occur in two representative periods, indexed by $t \in \{1, 2\}$. Each consumer demands at most one unit of the product throughout the selling season, and a consumer who purchases the product in period t derives a net utility $u_{it} = \delta^{t-1}(x_i q_i - p)$ (e.g., Desai 2001, Jerath et al. 2015). In the consumer’s utility function, x_i represents the consumer’s willingness-to-pay for quality (referred to as the consumer’s “type”), q_i is the quality experienced by the consumer after purchase, p is the product’s price, and δ is a discount factor that applies to second-period purchases. We assume that consumers are heterogeneous in their type, with x_i components inverse-uniformly distributed on the positive interval $[x_l, x_h] = [b^{-1}, a^{-1}]$; the corresponding probability density function is $g(x) = \frac{x^{-2}}{(b-a)}$, the cumulative distribution function is $G(x) = \frac{b-x^{-1}}{b-a}$, and we define $\bar{G}(x) := 1 - G(x)$.⁶ Consumers also differ in their ex post quality perceptions: conditional on the product’s underlying quality \hat{q} , we assume that a consumer’s post-purchase quality perception is a random draw from the distribution $q_i \sim N(\hat{q}, \sigma_i^2)$, where σ_i^2 measures the extent of heterogeneity in consumer-specific quality taste (e.g., Papanastasiou and Savva 2016).⁷ The discount factor $\delta \in [0, 1)$ captures the rate at which the product’s perceived value declines over time, but may also be interpreted as the degree of the consumers’ patience or as the level of strategic behavior in the consumer population.

In the first period, the consumers enter the market, observe the product’s design q , and form a rational prior belief over \hat{q} , which we denote by \tilde{q}_p ; since $\hat{q} = q + \epsilon$ and $\epsilon \sim N(0, \sigma_\epsilon^2)$, this prior belief is Normally distributed $\tilde{q}_p \sim N(q, \sigma_\epsilon^2)$.⁸ From the consumers who choose to purchase a unit in the first period, we assume that a fraction $\beta \in [0, 1]$ write a product review, with each review consisting of the consumer’s ex post perception of quality q_i .⁹ Parameter β is referred to throughout as the consumers’ “review propensity” and plays a central role in our analysis, as it essentially

⁵ While the quadratic cost function is convenient in terms of exposition, our main results hold for any twice-differentiable, increasing and convex cost function; wherever possible, we prove our results for such a function.

⁶ The inverse-uniform assumption is particularly useful in the derivation of analytical results in the presence of Bayesian learning; e.g., see Lemma 5 in Appendix A.

⁷ We note that this implies that a consumer’s type is independent of her ex post quality perception; this can be relaxed without loss, provided any association between the two is common knowledge.

⁸ Note that this is also the firm’s belief over \hat{q} at the beginning of the selling season. Therefore, there is no information asymmetry between the firm and the consumers, and the firm’s actions do not convey any additional information on product quality.

⁹ The analysis is unchanged if we assume, alternatively, that a review consists of the consumer’s ex post utility.

determines the pervasiveness of SL in the consumer population; the extreme case of $\beta = 0$ captures the benchmark setting where SL is absent.

In the second period, consumers remaining in the market engage in the review-based SL process described as follows. Let n denote the mass (henceforth the “number”) of customers who purchase in the first period, so that the number of reviews available in the second period is βn . Customers remaining in the market observe the reviews and update their belief over \hat{q} from \tilde{q}_p to \tilde{q}_u via Bayes’ rule. Given the above specifications of the consumers’ prior belief and the review-generating process, the updated belief is $\tilde{q}_u \sim N(q_u, \sigma_u^2)$, where

$$q_u = \frac{1}{n\beta\gamma + 1}q + \frac{n\beta\gamma}{n\beta\gamma + 1}R \text{ and } \sigma_u^2 = \frac{\sigma_\epsilon^2}{n\beta\gamma + 1}, \quad (1)$$

with R representing the average of first-period reviews and $\gamma = \frac{\sigma_\epsilon^2}{\sigma_i^2}$.^{10,11} In the consumers’ Bayesian update, the new quality expectation q_u is a convex combination of the prior expectation q and the average rating R . The weight placed by consumers on the average rating R increases with the number of product reviews, and with parameter γ , which is the ratio of prior quality uncertainty relative to review noise.

All of the above are common knowledge; in addition, each consumer has private knowledge of her own type, x_i . At the beginning of the selling season, the firm chooses a price-and-design policy (p, q) with the goal of maximizing its expected profit. For simplicity, we assume that the firm operates in the absence of any binding capacity constraints. The consumers make adoption decisions to maximize their expected utility; this implies, in particular, that a consumer will purchase the product in the first period only if the expected utility from doing so is nonnegative and greater than the expected utility of delaying the purchase decision.

Our analysis of the game between the firm and the consumers proceeds as follows. First, we consider the subgame played between the consumers under an arbitrary product policy (p, q) and establish the consumers’ purchase strategy. Then, we analyze the firm’s optimal price-and-design policy and investigate the implications of social learning for firm profit and consumer surplus. Throughout our analysis, we focus on equilibria in pure strategies.

4. Consumers’ Purchase Strategy

In the model described in §3, the presence of SL influences the consumers’ decision-making process in two respects. First, depending on the consumers’ level of patience δ , the anticipation of product reviews may affect their decision of *when* to make the purchase decision; second, for consumers who

¹⁰ Formally, since the purchase of each consumer is infinitesimally small, each review is only infinitesimally informative; thus, learning occurs on the aggregate of all available reviews (see Bergemann and Välimäki (1997) for more details).

¹¹ In our analysis, we treat γ and σ_ϵ as the two independent model parameters; σ_i is then specified as $\sigma_i^2 = \frac{\sigma_\epsilon^2}{\gamma}$.

remain in the market in the second period, the information contained in product reviews may affect their decision of *whether* to purchase the product. We begin our analysis with a characterization of these two effects, under an arbitrary firm policy (p, q) .

Consider first an individual consumer's decision of *when* to make her purchase decision. In our two-period model, each consumer decides in the first period whether to buy a unit or delay her purchase decision until the second period. According to the prior belief \tilde{q}_p , for a customer of type x_i the expected utility from purchasing in the first period is $x_i q - p$. On the other hand, if the consumer delays the decision, her expected utility from purchase in the second period will be $x_i q_u - p$, where q_u is the updated mean belief after observing the reviews of her peers; for the purposes of our model, q_u may be described as the "outcome of the SL process."

In deciding whether to delay her purchase decision until the second period, the consumer takes an expectation over her second-period utility with respect to the SL outcome, which is viewed in the first period as a random variable. Intuitively, a consumer's expected second-period utility increases with the amount of information (i.e., the number of reviews) available in the second period (see Lemma 4, Appendix A). However, notice that the amount of information available in the second period is endogenous to the adoption decisions of the consumers in the first period. The equilibrium in the consumers' adoption game is characterized by two opposing forces: on one hand, the higher the number of reviews consumers expect to be available in the second period, the higher the number of consumers who prefer to delay their purchase decision; on the other, the higher the number of consumers who delay their purchase decision, the smaller the number of reviews made available in the second period. Lemma 1 describes the unique threshold-type pure-strategy equilibrium that results. We use $\phi(\cdot)$ to denote the standard Normal pdf, with the corresponding cdf denoted by $\Phi(\cdot)$ and $\bar{\Phi}(\cdot) := 1 - \Phi(\cdot)$.

LEMMA 1. *For any given (p, q) , $\frac{p}{q} \in [x_l, x_h]$, a consumer of type x_i will purchase a unit in the first period provided $x_i \geq z(p, q; \delta)$, where $z(p, q; \delta) \in \left[\frac{p}{q}, x_h\right]$ is the unique solution to the implicit equation*

$$Y + \delta \int_Y^\infty \bar{\Phi}(s) ds = 0,$$

with $Y = \frac{p-q}{\sigma_z}$ and $\sigma_z = \sigma_\epsilon \sqrt{\frac{G(z)\beta\gamma}{G(z)\beta\gamma+1}}$. Furthermore, z is increasing in the review propensity β , in the consumers' patience level δ , and in the product's price p , and decreasing in the product's expected quality q .

All proofs are provided in the Appendix. According to Lemma 1, higher-type consumers purchase in the first period in order to avoid discounted utility, while lower-type consumers prefer to wait until they are more informed. Notice that the purchasing threshold z is higher than $\frac{p}{q}$, which

suggests informational free-riding: at least some consumers whose expected utility from purchase is positive in the first period nevertheless choose to delay adoption in anticipation of the reviews of their peers. The properties associated with the threshold strategy are intuitive: if the first-period buyers' tendency to write reviews is higher, and/or the consumers are more patient, and/or the product's price is higher, a relatively larger number of consumers decide to delay their purchase decision; on the other hand, if the product's ex ante expected quality is higher, fewer consumers find the wait for more information to be worthwhile.

In the second period, the decision of consumers remaining in the market is straightforward: they simply observe the reviews of first-period buyers, update their belief over product quality to \tilde{q}_u , and purchase a unit provided $x_i q_u - p \geq 0$.

5. Firm's Policy Choice

Building on the consumers' purchase equilibrium described in §4, we now analyze the firm's optimal price-and-design policy. The firm's expected profit at policy (p, q) can be written as

$$\pi(p, q) = (p - c(q)) \left(\int_z^{x_h} g(x) dx + \int_{\frac{p}{z}}^{\infty} \int_{\max\{\frac{p}{s}, x_l\}}^z g(x) f(s; q, z) dx ds \right), \quad (2)$$

where z is the consumers' first-period purchase threshold (see Lemma 1), and $f(\cdot; q, z)$ is the pdf of the consumers' updated mean belief after observing the reviews of their peers (which in the first period is viewed as a random variable); the ex ante distribution of the ex post mean belief is often referred to in the literature as the *preposterior distribution*, a term which we henceforth adopt. In Lemma 4, Appendix A, we show that $f(\cdot; q, z)$ is a Normal pdf with mean q and standard deviation¹²

$$\sigma_z = \sigma_\epsilon \sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma + 1}}. \quad (3)$$

In the firm's profit function, the first parenthesis captures the firm's profit per unit sold, while the second parenthesis captures its total expected sales. In the first period, consumers of type $x_i \geq z$ purchase a unit, resulting in a first-period sales volume of $\int_z^{x_h} g(x) dx$. In the second period, any sales are contingent on the content of product reviews: no sales occur if the updated mean belief s is lower than the threshold $\frac{p}{z}$ (i.e., when the outcome of SL is “negative”), while in the opposite case (i.e., when the outcome is “positive”) the number of sales achieved increases with the social learning outcome s up until the entire market is captured.

¹² For a more thorough discussion of the preposterior distribution and its properties, see §4 of Papanastasiou and Savva (2016).

5.1. Benchmark: Policy Choice in the Absence of Social Learning

We begin our analysis of the firm's optimal policy by establishing a benchmark policy which is optimal in the absence of SL. The benchmark setting is retrieved from the general model by setting $\beta = 0$, such that no consumers engage in writing reviews. We point out that in the absence of SL, the consumers can only resolve their quality uncertainty if they purchase and experience the product themselves. Since there is no opportunity to learn from their peers, there is also no reason for consumers to delay their purchase decision until the second period. Therefore, the model collapses to a single-period model where, given the firm's policy choice, only consumers with expected utility $x_i q - p \geq 0$ purchase a unit. The firm's profit, as a function of its price-and-design choice (p, q) , is given simply by

$$\pi_0(p, q) = (p - c(q)) \int_{\frac{p}{q}}^{x_h} g(x) dx. \quad (4)$$

In identifying the optimal policy, the firm faces a tradeoff involving the profit margin $(p - c(q))$, and the volume of sales achieved $\int_{\frac{p}{q}}^{x_h} g(x) dx$.

LEMMA 2. *Suppose that the consumers' type distribution satisfies $2a \leq b$. In the absence of SL, the firm's optimal policy is given by*

$$p_0^* = \frac{1}{8a^2} \text{ and } q_0^* = \frac{1}{4a}.$$

The condition $2a \leq b$ ensures that the firm's optimal policy is "interior," in the sense that the firm does not find it optimal to choose a price-and-design combination that covers the entire market. Since the goal of our analysis below is to identify how the SL process causes the firm to adjust its policy, in the subsequent sections we employ this condition as an assumption.

ASSUMPTION 1. *The consumers' type distribution satisfies $2a \leq b$.*

5.2. Policy Choice in the Presence of Social Learning

We now return to the general model, where consumers remaining in the market in the second period update their quality beliefs and make purchase decisions on the basis of the first-period buyers' reviews, and the firm optimizes its policy in the presence of SL.

We consider first the simpler case where the consumers are myopic ($\delta = 0$), so that no consumer delays her purchase decision in anticipation of product reviews. In this case, the first-period purchase threshold in (2) reduces to $z = \frac{p}{q}$, and the optimal policy is described as follows.

PROPOSITION 1. *When the consumers are myopic:*

- (i) *If the ex ante quality uncertainty is low (i.e., for sufficiently small σ_ϵ), the firm's optimal policy (p_m^*, q_m^*) satisfies $p_m^* < p_0^*$, $q_m^* < q_0^*$, and $\frac{p_m^*}{q_m^*} < \frac{p_0^*}{q_0^*}$.*

(ii) If the *ex ante* quality uncertainty is high (i.e., for sufficiently large σ_ϵ), the firm's optimal policy (p_m^*, q_m^*) satisfies $p_m^* > p_0^*$, $q_m^* > q_0^*$, and $\frac{p_m^*}{q_m^*} > \frac{p_0^*}{q_0^*}$.

Proposition 1 reveals an interesting phenomenon: the firm's reaction to the presence of SL depends critically on the magnitude of the quality uncertainty surrounding the product, with the two extremes of low and high uncertainty calling for diametrically opposite policy choices. The result is illustrated in Figure 1. Observe that as the quality uncertainty increases from zero, the firm initially opts for a product of inferior design accompanied by a lower price; however, as the uncertainty continues to increase, the firm gradually shifts its preference towards a superior design accompanied by a higher price. Notice also that across all values of quality uncertainty, the firm opts for a product of lower expected quality either in the absolute sense (i.e., $q_m^* < q_0^*$), or relative to the product's price (i.e., $\frac{p_m^*}{q_m^*} > \frac{p_0^*}{q_0^*}$); indeed, in Lemma 7, Appendix A, we establish this feature of the firm's optimal policy analytically.

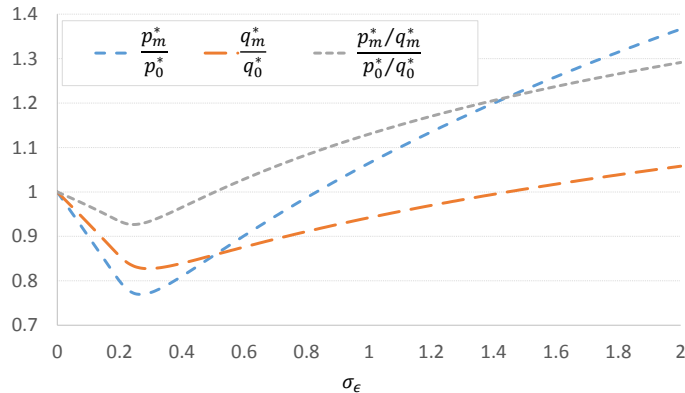


Figure 1 Optimal price-and-design policy as a function of quality uncertainty σ_ϵ , when the consumers are myopic. Parameter values: $a = 0.5$, $b = 2$, $\beta = \gamma = 1$.

To explain the result of Proposition 1, we begin by pointing out that any policy that departs from (p_0^*, q_0^*) entails trading off profit margin in favor of increased first-period adoption, or vice-versa (to see this, note that if a policy existed that simultaneously increased both profit margin and first-period adoption, this policy would be preferred to (p_0^*, q_0^*) in Lemma 2). With this in mind, observe that in Proposition 1(i), the firm opts for increased early adoption (and hence a decreased profit margin) by offering consumers a lower *price per unit of quality*; by contrast, in Proposition 1(ii) the firm takes the opposite approach. The rationale underlying this dichotomy in the firm's approach with respect to the magnitude of the quality uncertainty σ_ϵ lies in the relative importance of early product adoption in driving the SL process, and therefore the firm's total expected sales.

To understand how this importance depends on σ_ϵ , note first that the impact of SL on the firm's total expected sales is captured through the preposterior distribution f (see (2)), and more specifically through this distribution's standard deviation, σ_z (see (3)). It is useful to think of σ_z as the *extent of consumer learning*, as it is essentially a measure of the extent by which consumers' valuations for the product are likely to change from the first period to the second. For the sake of building intuition, we may momentarily consider σ_z as an exogenous quantity, and note that the firm's total expected sales are increasing and concave in the magnitude of σ_z . Thus, all else being equal, the firm's expected sales are more sensitive to changes in σ_z when these changes occur around smaller magnitudes.

Returning to the model where the extent of consumer learning σ_z is endogenously determined through the firm's policy, we first point out that σ_z increases with the volume of reviews generated in the first period, which itself increases with the volume of early adoption. However, we note also that σ_z is upper-bounded by the magnitude of the ex ante quality uncertainty σ_ϵ . When σ_ϵ is relatively small, this implies that the magnitude of σ_z is also small. Therefore, the firm in this case operates in a region of consumer learning where total expected sales are highly sensitive to the volume of first-period purchases: if the firm moves to increase (decrease) early adoption, total expected sales will increase (decrease) significantly. As a result, the firm opts for a decreased profit margin and an increase in early adoption; as Proposition 1(i) suggests, the most efficient way of achieving this is through a reduction in both the price and expected quality.

Consider next the opposite case, where the ex ante uncertainty σ_ϵ is large. In this case, the policy (p_0^*, q_0^*) which is employed by the firm in the absence of SL, along with the first-period adoption that this policy entails, places the firm in a region of consumer learning where the total expected sales are relatively insensitive to early product adoption – intuitively, when uncertainty is high, even a small number of reviews is sufficient to change consumers' valuations to a great extent. In turn, the firm, recognizing that a reduction in early adoption does not significantly impact its total expected sales, chooses to focus instead on identifying policies where early adoption is lower but profit margins are higher; Proposition 1(ii) suggests that among all such policies, those that combine high expected quality with high price are the most profitable.

The dichotomy in the firm's approach regarding the optimal price per unit of quality as described in Proposition 1 is particularly interesting in light of the following observation.

COROLLARY 1. *Suppose that either the price or the design of the product are exogenously fixed. Then the optimal price per unit of quality is always higher in the presence of SL.*

In particular, holding the price (respectively, design) of the product constant, the firm will always choose a product of inferior design (higher price) in the presence of SL; as a result, the optimal

price per unit of quality is always higher. Thus, we observe that the firm’s decision to decrease the price per unit of quality at low values of σ_ϵ in Proposition 1 arises only when the firm has the ability to maintain balance with respect to its profit margin by adjusting both price and design simultaneously.

We consider next the firm’s optimal policy when the consumers are, at least to some degree, strategic ($\delta > 0$). In this case, the first-period purchasing threshold z in (2) is described in Lemma 1. Proposition 2 below focuses on how the presence of strategic consumer behavior affects the firm’s optimal price-and-design combination.

PROPOSITION 2. *When the consumers are strategic with $\delta \geq \Delta$, for some $\Delta \in (0, 1)$:*

- (i) *If the ex ante quality uncertainty is low (i.e., for sufficiently small σ_ϵ), the firm’s optimal policy satisfies $p_s^* > p_m^*$, $q_s^* > q_m^*$, and $\frac{p_s^*}{q_s^*} > \frac{p_m^*}{q_m^*}$.*
- (ii) *If the ex ante quality uncertainty is high (i.e., for sufficiently large σ_ϵ), the firm’s optimal policy satisfies $p_s^* < p_m^*$, $q_s^* < q_m^*$, and $\frac{p_s^*}{q_s^*} < \frac{p_m^*}{q_m^*}$.*

Recall that when facing myopic consumers, in cases of low (high) quality uncertainty the firm elected to decrease (increase) the product’s price, quality and price per unit of quality. Proposition 2 suggests that strategic consumer behavior effectively causes the firm to “roll back” its policy adjustments; indeed, in the example of Figure 2 below, we observe that the firm’s policy is qualitatively similar as that in the absence of SL (i.e., in comparison to Figure 1), albeit with policy adjustments that are less pronounced. This is not surprising: in the case of myopic consumers, the firm’s policy adjustments were geared towards taking advantage of the SL process; however, as consumers become more strategic, fewer consumers are willing to purchase in the first period, rendering the impact of SL on the firm’s expected sales weaker —and this is true irrespective of the firm’s choice of policy. As a result, the firm gradually reverts back to a policy that ignores the presence of SL, which carries the characteristics described in Proposition 2.

6. Firm Profit

We now consider how the presence of SL affects the firm’s expected profit. As in the previous section, we start with the simpler case of myopic consumers. When the consumers are myopic, the firm’s sales in the first period at policy (p, q) are identical to those in the absence of SL (see §5.1), while in the second period, the firm achieves additional sales in the event that the reviews of first-period consumers are favorable. Proposition 3 establishes that the higher the proportion of consumers who engage in writing reviews, the higher the firm’s expected profit.

PROPOSITION 3. *When the consumers are myopic, the firm’s optimal expected profit is monotonically increasing in the consumers’ review propensity β . That is, the presence of SL is ex ante beneficial for the firm.*

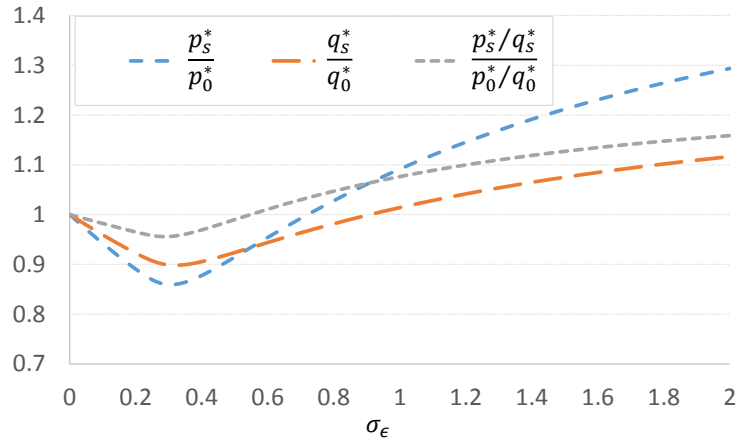


Figure 2 Optimal price-and-design policy as a function of quality uncertainty σ_ϵ , when the consumers are strategic. Parameter values: $a = 0.5$, $b = 2$, $\beta = \gamma = 1$, $\delta = 0.6$.

To draw the conclusion of Proposition 3, an argument based on a fixed policy will suffice. For any fixed price and design, as the consumers' review propensity β increases, the firm's profit in the first period remains unchanged, but the extent of consumer learning in the second period, captured by σ_z , increases. The increase in the variance of the preposterior distribution renders both more positive and more negative second-period learning outcomes ex ante more likely; however, the effect of this variability on the firm's expected sales is asymmetrically positive: while the firm benefits as more positive outcomes become more likely, it does not suffer as more negative outcomes become more likely, since all negative outcomes result in equivalent (i.e., zero) second-period sales. It follows that as β increases the firm's profit can only increase further if the firm chooses price and design optimally.

Thus, when the consumers are myopic, the impact of SL on firm profit is always positive. However, when the consumers are strategic, the presence of SL is also accompanied by two detrimental effects. The first is the "delay effect," which refers to the potential loss of sales from consumers who, despite having a positive expected utility from purchase in the first period, decide to delay their decision until the second period (i.e., consumers of type $\frac{p}{q} < x_i \leq z$ in our model). All such sales are retained by the firm only if the updated quality belief s is higher than the prior q ; if the opposite occurs, either a fraction (moderately lower s) or all (significantly lower s) of these sales are lost. The second is the "decreased learning effect," which is more subtle but equally significant. This effect refers to the expected loss of sales that occurs as a result of a weaker SL effect. In particular, it is straightforward to show that, all else being equal, the firm prefers a higher volume of reviews in the second period, because a higher review volume exerts a stronger influence on the

valuations of consumers remaining in the market. However, because of the strategic purchasing delays of consumers in the first period, the volume of reviews available in the second period is decreased, resulting in a loss of expected sales generated from consumers remaining in the market in the second period (i.e., consumers of type $x_i < \frac{p}{q}$ in our model).

Thus, strategic purchase delays cause “market shrinkage” both directly in the first period, through intertemporal purchase-decision shifts, and indirectly in the second period, through a weaker SL effect. The interesting question is whether the negative effects that arise from the interaction between SL and strategic consumer behavior can be potent enough to render the presence of SL detrimental altogether.

PROPOSITION 4. Suppose that the consumers’ type distribution satisfies $3a > b$. When the consumers are strategic with sufficiently high δ , the firm’s optimal expected profit is monotonically decreasing in the consumers’ review propensity β . That is, the presence of SL is ex ante detrimental for the firm.

Proposition 4 suggests that when the consumers are highly strategic, the negative effects of strategic consumer behavior can be strong enough to render the presence of SL ex ante detrimental for the firm. To explain and illustrate this phenomenon, we enlist the example of Figure 3, where we plot the firm’s optimal profit as a function of the consumers’ review propensity β . Observe that for intermediate-to-high values of δ , the firm’s profit is nonmonotone in β , initially increasing and then decreasing. From Proposition 3, we know that in the absence of strategic purchasing delays, the firm’s profit increases with β ; however, note that according to Lemma 1 the consumers’ tendency to strategically delay purchase also increases with β . The numerical experiments of Figure 3 illustrate that the positive effects of consumer learning tend to dominate the negative effects of strategic consumer behavior at low values of β , but that the opposite is true for higher values of β . As the consumers’ patience δ increases, the range of values of β for which the firm benefits from SL shrinks from above and eventually vanishes, leading to the result of Proposition 4.

To conclude this section, it is instructive to discuss how the results of Proposition 3 and 4 relate to findings in the existing literature. The interaction between SL from product reviews and strategic consumer behavior is considered in two recent papers by Papanastasiou and Savva (2016) and Yu et al. (2015), which find that SL is ex ante beneficial for the firm when the consumers are myopic, and continues to be beneficial even once the detrimental effects of strategic consumer behavior are accounted for.¹³ Our Proposition 3 echoes the conclusion pertaining to the case of myopic consumers; however, Proposition 4 suggests that when the consumers are strategic, the

¹³ Yu et al. (2015) present an example where the firm is worse off in the presence of SL; however, in that example the firm is assumed to be less strategic than the consumers.

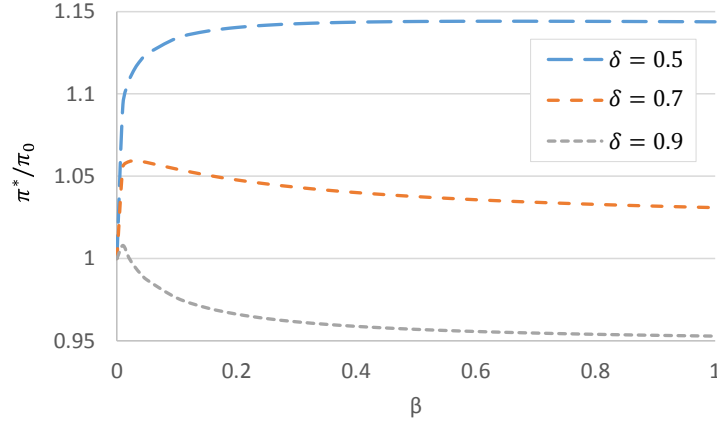


Figure 3 Expected firm profit as a function of consumers' review propensity β , at different values of consumer patience δ . Parameter values: $a = 0.9$, $b = 2$, $\sigma_\epsilon = \gamma = 1$.

picture is less clear.¹⁴ The key difference between our model and those of Papanastasiou and Savva (2016) and Yu et al. (2015) is the mode of consumer heterogeneity considered: in this paper, consumers are heterogeneous in the vertical sense (i.e., consumers differ in their willingness-to-pay for the product's quality attributes), while in theirs consumers are heterogeneous in their horizontal sense (i.e., consumers differ in their willingness-to-pay for the product's search attributes). As our understanding of the interaction between SL and strategic consumer behavior continues to grow, Proposition 4 serves as a caution that such results may not hold outside the specific settings to which they correspond.

7. Consumer Surplus

So far, our analysis has focused on the implications of SL for the firm. In this section, attention is focused on the consumers' perspective, which becomes especially interesting in light of the preceding analysis. On one hand, SL allows consumers to learn from the experiences of their peers, and as a result, to make better-informed purchase decisions (it is straightforward to show that, all else being equal, SL results in an increase in expected consumer surplus); on the other, however, the analysis of §5 suggests that as a result of the consumers' ability to learn from their peers, the firm may opt for a product of lower expected quality and/or charge a higher price per unit of quality, which may negatively impact consumer surplus. The magnitude of either effect is complicated by the discount factor δ applied to second-period utility and the associated strategic purchasing delays this generates.

To understand the impact of SL on the consumers' expected surplus, it is instructive to consider how consumers at opposite ends of the type spectrum are impacted by its presence. Consumers at

¹⁴ We note that in the proof of Proposition 4, we show that the result holds unchanged when either the product's price or its design are exogenously fixed.

the lower end (i.e., low- x_i consumers) are typically consumers who would not purchase the product in the absence of SL. These consumers are the primary beneficiaries of the SL, as the following result suggests.

LEMMA 3. *For any $\delta \in (0, 1)$, there exists a threshold type $\psi \in [\frac{1}{2a}, \frac{1}{a}]$ such that every consumer of type $x_i \leq \psi$ has strictly higher expected surplus in the presence of SL.*

Consider, for instance, a consumer of the lowest type (i.e., $x_i = b^{-1}$). In the absence of SL, this consumer would never purchase a unit, but in its presence, if the consumer learns in the second period that the product's quality is much higher than she originally expected, she may then decide to purchase. Thus, while in the absence of SL her expected surplus is zero, in its presence it is strictly positive ($\delta \int_{bp}^{\infty} (\frac{s}{b} - p) f(s; q, z) ds > 0$), and this is true irrespective of the firm's chosen policy.

But what about consumers at the higher end of the type spectrum (i.e., high- x_i consumers)? These are consumers who are typically early adopters of the product. Since these consumers tend not to stay in the market long enough to benefit from the reviews of other consumers, the main impact that SL has on them is through the firm's choice of product policy. Unfortunately for these consumers, the firm's policy in the presence of SL tends to be one that results in a surplus decrease; the proposition that follows highlights this effect.

PROPOSITION 5. *When the consumers are myopic:*

(i) *If the ex ante quality uncertainty is low (i.e., for sufficiently small σ_ϵ), the consumers' expected surplus is higher in the presence of SL if and only if*

$$\beta < \left(\frac{3 - 4 \ln 2}{2 \ln 2 - 1} \right) \left(\frac{b - a}{a\gamma} \right).$$

(ii) *If the ex ante uncertainty is high (i.e., for sufficiently large σ_ϵ), the consumers' expected surplus is lower in the presence of SL.*

In Proposition 5, only the expected surplus of first-period buyers contributes to total expected surplus, since in the case of myopic consumers the utility of second-period consumers is completely discounted. Thus, the result is dominated by the impact of SL on the firm's policy choice, and how this affects early adopters of the product. The result suggests that SL benefits these consumers only if the ex ante quality uncertainty is low *and* not many consumers tend to write reviews. When the quality uncertainty is low, recall from Proposition 1 that the firm focuses on increasing early adoption, by reducing both price and quality. Because the consumers are heterogeneous in their quality preferences, the overall impact of such a policy depends on the relative decrease of price versus quality. As Proposition 5 suggests, this is favorable for the consumers only when their review propensity is low. On the other hand, when the uncertainty is high, the firm focuses on maximizing

its profit margin by increasing price and quality; in turn this implies a decrease in early product adoption, which results in a decrease in the consumers' surplus. Thus, the overall pattern that emerges is that early adopters are likely to benefit from SL if and only if consumer learning is limited, either because there is no significant uncertainty to begin with, or because consumers tend not to engage in writing reviews.

In Figure 4, we plot the consumers' expected surplus against the consumers' discount factor δ , for three different values of review propensity β . Observe first that as the consumers become more strategic, their expected surplus increases. At low values of δ , the impact of SL on the consumers' surplus is dominated by the effect of the firm's policy adjustment on the surplus of early adopters, which, as discussed above, tends to be negative. By contrast, at high values of δ , the majority of consumers delay adoption and the positive learning effect of SL tends to dominate. This also leads to the further observation that a higher review propensity is beneficial for the consumers only provided they are sufficiently strategic. In particular, as Proposition 5 suggests, early adopters are better off when the consumers' review propensity is low; it follows that when consumer surplus is dominated by the impact of SL on early adopters (i.e., at low values of δ), a lower review propensity is advantageous. By contrast, when consumer surplus is dominated by the positive learning effect of SL (i.e., at high values of δ), a higher review propensity that strengthens the learning process is beneficial.

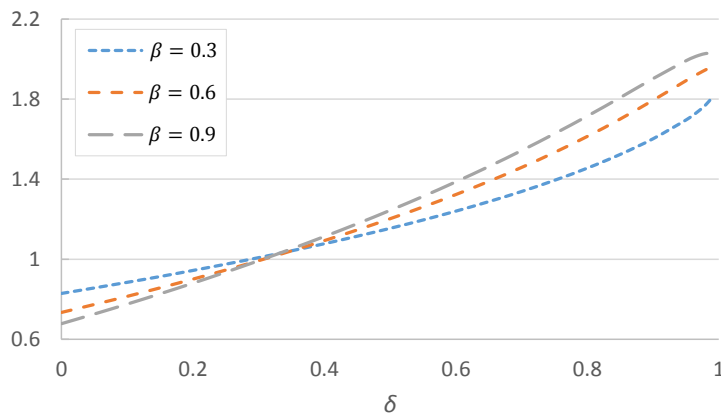


Figure 4 Expected consumer surplus as a function of consumer patience δ , at different values of review propensity β . Parameter values: $a = 0.5$, $b = 2$, $\sigma_e = \gamma = 1$.

8. Discussion

This paper studies the interaction between review-based SL and a monopolist firm's choice of product price and design. The analysis highlights that the firm will employ different policies depending

on the level of the consumers' ex ante quality uncertainty: in environments of low (high) uncertainty, the firm chooses a product of inferior (superior) design accompanied by a low (high) price. This finding offers a new perspective on the commonly held notion that SL should push firms to invest more in the quality of their products.¹⁵ In particular, our analysis suggests that this notion can be incorrect (i.e., the firm does not necessarily increase its investment in quality), or incomplete (i.e., even when the firm does increase its quality investment, it is often at the expense of the consumers, who are charged a disproportionately high price). We also observe that, contrary to conventional wisdom, the presence of SL does not necessarily leave the firm better off ex ante, especially when consumers are highly strategic in the timing of their purchase decisions; conversely, we observe that consumers tend to be better off in the presence of SL only provided that they behave strategically.

Our model makes several simplifying assumptions which represent potential avenues for future research. For instance, the current model focuses on the case where the firm and the consumers are ex ante symmetrically informed about the product's realized quality (i.e., conditional on the firm's observable design choice). In reality, however, we might expect the firm to become better informed regarding the quality of its product as the start of the selling season approaches (e.g., through market research, focus groups, product testing, etc.). To model such cases, we might assume that the firm first chooses the product design, receives an informative signal regarding the realized product quality, and then chooses the product price at the beginning of the selling season. Two interesting questions arise. First, how does the firm's design choice take into account the opportunity to learn about the realized quality before setting the price? Second, to what extent does the firm's pricing decision reveal product quality to the consumers?

Another simplification in our modeling approach is the assumption of fixed pricing. Here, we might ask how the firm's design choice would differ from that in our model under the assumption of dynamic pricing. Under such a pricing scheme (but under the assumption of a fixed product design) Yu et al. (2015) identify the cases in which the firm in the presence of SL will choose to increase/decrease early adoption in order to reinforce/dampen the SL process (through its choice of introductory price). It would be of interest to see whether and to what extent the conclusions regarding early adoption hold in the case of endogenous design, and how in implementing them the firm adjusts the product's price and design. Finally, we may consider the interaction between production capacity constraints and the firm's optimal policy. Constraints in supply tend to counteract the consumers' tendency to delay their purchase decision. Thus, we might expect that in

¹⁵ In the words of *Amazon.com* CEO Jeff Bezos: "The individual is getting empowered. And [...] the right way to respond to this if you're a company is to say, 'OK, I'm going to put the vast majority of my energy, attention and dollars into building a great product or service [...] because I know if I build a great product or service, my customers will tell each other'" (Rose 2010).

the presence of supply constraints, the firm can be more aggressive in its policy choice, either by reducing the expected quality of the product to cut costs, and/or by charging a higher product price.

Appendix

A. Supporting Results

LEMMA 4. *Suppose that m reviews become available to consumers remaining in the market in the second period. Then, in the first period, the expected utility from delaying purchase for a customer of type x_i is*

$$E[u_{i2}] = \delta \int_{\frac{p}{x_i}}^{\infty} (x_i s - p) h(s; q, m) ds, \quad (5)$$

where $h(\cdot; q, m)$ is a Normal density function with mean q and standard deviation $\sigma_\epsilon \sqrt{\frac{m\gamma}{m\gamma+1}}$. Furthermore, $E[u_{i2}]$ is strictly increasing in m .

Proof. If m product reviews become available in the second period and their mean is R , then by Bayes' rule, the posterior belief \tilde{q}_u is Normally distributed with a mean of

$$q_u = \frac{\sigma_i^2}{m\sigma_\epsilon^2 + \sigma_i^2} q + \frac{m\sigma_\epsilon^2}{m\sigma_\epsilon^2 + \sigma_i^2} R = \frac{1}{m\gamma+1} q + \frac{m\gamma}{m\gamma+1} R,$$

where q is the mean of the prior belief and $\gamma = \frac{\sigma_i^2}{\sigma_\epsilon^2}$. The posterior mean belief q_u is viewed in the first period as a random variable (r.v.), since it depends on the unobservable realization of quality \hat{q} , as well as the (noisy) average of first period reviews. Conditional on product quality \hat{q} , the sample mean of m (i.i.d. Normal) reviews, R , follows $R \sim N(\hat{q}, \frac{\sigma_i^2}{m})$ and the updated mean belief follows $q_u | \hat{q} \sim N\left(\frac{1}{m\gamma+1} q + \frac{m\gamma}{m\gamma+1} \hat{q}, \left(\frac{m\gamma}{m\gamma+1}\right)^2 \frac{\sigma_i^2}{m}\right)$. Next, as \hat{q} is an ex ante Normal r.v. ($\hat{q} \sim N(q, \sigma_\epsilon^2)$), we have

$$\begin{aligned} E[q_u] &= E(E[q_u | \hat{q}]) = E\left(\frac{1}{m\gamma+1} q + \frac{m\gamma}{m\gamma+1} \hat{q}\right) = q, \text{ and} \\ \text{Var}[q_u] &= E(\text{Var}[q_u | \hat{q}]) + \text{Var}(E[q_u | \hat{q}]) \\ &= \left(\frac{m\gamma}{m\gamma+1}\right)^2 \left(\frac{\sigma_i^2}{m} + \sigma_\epsilon^2\right) = \left(\frac{m\gamma}{m\gamma+1}\right)^2 \left(\frac{\sigma_i^2(m\gamma+1)}{m}\right) = \frac{m\gamma}{m\gamma+1} \sigma_\epsilon^2. \end{aligned}$$

Therefore, the updated mean belief q_u , upon which the consumer bases her purchase decision in the second period, is viewed in the first period as a Normal r.v. with mean q and variance $\frac{m\gamma}{m\gamma+1} \sigma_\epsilon^2$; let $h(\cdot; q, m)$ denote the corresponding pdf. In the second period, a consumer of type x_i will purchase only if her updated expected utility from purchase is positive. Thus, in the first period her expected utility from delaying purchase is

$$E[u_{i2}] = \delta \int_{\frac{p}{x_i}}^{\infty} (x_i s - p) h(s; q, m) ds.$$

Now, note that an increase in m increases the variance of the preposterior distribution of q_u , without changing its mean. Since (i) the consumer's second-period expected utility is nonnegative, increasing and convex in q_u and (ii) q_u is a Normal r.v., it follows from second-order stochastic dominance that the consumer's utility from delaying purchase is increasing in m . \square

LEMMA 5. *The firm's profit function can be written as*

$$\pi(p, q) = \frac{p - c(q)}{b - a} \left(\frac{q}{p} - a + \frac{\sigma_z}{p} (\phi(Y) - \phi(V) + Y\Phi(Y) + V\bar{\Phi}(V)) \right),$$

where $V = \frac{bp - q}{\sigma_z}$, $Y = \frac{pz^{-1} - q}{\sigma_z}$, z is given in Proposition 1, and σ_z is given in (3).

Proof. In the general model with SL, the firm's profit function when consumers of type $x_i \geq z$ purchase in the first period is given by

$$\pi(p, q) = (p - c(q)) \int_z^{x_h} g(x) dx + (p - c(q)) \int_z^\infty \int_{\max\{\frac{p}{s}, x_i\}}^z g(x) f(s; q, z) dx ds.$$

Since $g(\cdot)$ is an inverse-uniform pdf, we may rewrite the above as

$$\begin{aligned} \pi(p, q) &= (p - c(q)) \frac{z^{-1} - a}{b - a} + (p - c(q)) \int_z^\infty \min\left(\frac{\frac{s}{p} - z^{-1}}{b - a}, \frac{b - z^{-1}}{b - a}\right) f(s; q, z) dx ds \\ &= \frac{(p - c(q))}{b - a} \left(z^{-1} - a + \int_z^{bp} \left(\frac{s}{p} - z^{-1}\right) f(s; q, z) ds + \int_{bp}^\infty (b - z^{-1}) f(s; q, z) ds \right) \end{aligned}$$

Let $V = \frac{bp - q}{\sigma_z}$ and $Y = \frac{pz^{-1} - q}{\sigma_z}$. Then

$$\begin{aligned} \pi(p, q) &= \frac{p - c(q)}{b - a} \left(z^{-1} - a + \frac{\sigma_z}{p} (\phi(Y) - \phi(V)) + \left(\frac{q}{p} - z^{-1}\right) (\Phi(V) - \Phi(Y)) + (b - z^{-1}) \bar{\Phi}(V) \right) \\ &= \frac{p - c(q)}{b - a} \left(\frac{q}{p} - a + \frac{\sigma_z}{p} (\phi(Y) - \phi(V) + Y\Phi(Y) + V\bar{\Phi}(V)) \right). \square \end{aligned}$$

LEMMA 6. *Let*

$$S(p, q) = \frac{1}{b - a} \left(\frac{q}{p} - a + \frac{\sigma_z}{p} (\phi(Y) - \phi(V) + Y\Phi(Y) + V\bar{\Phi}(V)) \right)$$

denote the firm's total expected sales, where $V = \frac{bp - q}{\sigma_z}$, $Y = \frac{pz^{-1} - q}{\sigma_z}$, z is given in Proposition 1, and σ_z is given in (3). Let S_p and S_q denote the partial derivatives of S with respect to p and q respectively. The firm's optimal policy satisfies

$$c'(q) = -\frac{S_q}{S_p}, \tag{6}$$

where $c(q)$ is the firm's cost function.

Proof. In the general model with SL, the firm's profit function is

$$\pi(p, q) = (p - c(q))S(p, q).$$

The first-order necessary conditions for optimality are

$$\pi_p = S + (p - c(q))S_p = 0 \tag{7}$$

$$\pi_q = -c'(q)S + (p - c(q))S_q = 0 \tag{8}$$

From (8) we have $S = \frac{(p - c(q))S_q}{c'(q)}$. Inserting into (7), we have $c'(q) = -\frac{S_q}{S_p}$. \square

LEMMA 7. *When the consumers are myopic, if the firm's policy satisfies $q_m^* \geq q_0^*$, then it also satisfies*

$$\frac{p_m^*}{q_m^*} > \frac{p_0^*}{q_0^*}.$$

Proof. From Lemma 6 we know that the optimal policy must satisfy $c'(q) = -\frac{S_q}{S_p}$. When the consumers are myopic, we have

$$S(p, q) = \frac{1}{b-a} \left(\frac{q}{p} - a + \frac{\sigma_z}{p} [\phi(0) - \phi(V) + V\bar{\Phi}(V)] \right),$$

for $V = \frac{bp-q}{\sigma_z} \geq 0$, $z = \frac{p}{q}$. The partial derivatives of S are

$$\begin{aligned} S_p &= \frac{1}{b-a} \left(-\frac{q}{p^2} + \frac{\partial}{\partial p} \left(\frac{\sigma_z}{p} \right) [\phi(0) - \phi(V) + V\bar{\Phi}(V)] + \frac{\sigma_z}{p} V_p \bar{\Phi}(V) \right) \\ &= \frac{1}{p(b-a)} \left(-\frac{q}{p} \Phi(V) + \left(\frac{\partial \sigma_z}{\partial p} - \frac{\sigma_z}{p} \right) [\phi(0) - \phi(V)] \right) \\ S_q &= \frac{1}{b-a} \left(\frac{1}{p} + \frac{\partial}{\partial p} \left(\frac{\sigma_z}{p} \right) [\phi(0) - \phi(V) + V\bar{\Phi}(V)] + \frac{\sigma_z}{p} V_q \bar{\Phi}(V) \right) \\ &= \frac{1}{p(b-a)} \left(\Phi(V) + \frac{\partial \sigma_z}{\partial q} [\phi(0) - \phi(V)] \right) \end{aligned}$$

Next, we note that $\frac{\partial \sigma_z}{\partial p} = \frac{\partial \sigma_z}{\partial z} \frac{\partial z}{\partial p} = \frac{\partial \sigma_z}{\partial z} \frac{1}{q} = -\frac{q}{p} \frac{\partial z}{\partial q} \frac{\partial \sigma_z}{\partial z} = -\frac{q}{p} \frac{\partial \sigma_z}{\partial q}$ so that inserting into $c'(q) = -\frac{S_q}{S_p}$ we have

$$\begin{aligned} c'(q) &= -\frac{S'_q}{S'_p} = -\frac{\Phi(V) + \frac{\partial \sigma_z}{\partial q} [\phi(0) - \phi(V)]}{-\frac{q}{p} \Phi(V) + \left(\frac{\partial \sigma_z}{\partial p} - \frac{\sigma_z}{p} \right) [\phi(0) - \phi(V)]} = \frac{\Phi(V) + \frac{\partial \sigma_z}{\partial q} [\phi(0) - \phi(V)]}{\frac{q}{p} \Phi(V) + \left(\frac{q}{p} \frac{\partial \sigma_z}{\partial q} + \frac{\sigma_z}{p} \right) [\phi(0) - \phi(V)]}, \\ \frac{1}{c'(q)} &= \frac{\frac{q}{p} \left(\Phi(V) + \frac{\partial \sigma_z}{\partial q} [\phi(0) - \phi(V)] \right) + \frac{\sigma_z}{p} [\phi(0) - \phi(V)]}{\Phi(V) + \frac{\partial \sigma_z}{\partial q} [\phi(0) - \phi(V)]} = \frac{q}{p} + \frac{\frac{\sigma_z}{p} [\phi(0) - \phi(V)]}{\Phi(V) + \frac{\partial \sigma_z}{\partial q} [\phi(0) - \phi(V)]}. \end{aligned}$$

Multiplying through by $\frac{p}{q}$ we have

$$\frac{p}{qc'(q)} = 1 + \frac{\frac{\sigma_z}{q} [\phi(0) - \phi(V)]}{\Phi(V) + \frac{\partial \sigma_z}{\partial q} [\phi(0) - \phi(V)]}$$

Now, notice that the last term is positive since $\frac{\partial \sigma_z}{\partial q} > 0$. Therefore, we have that the optimal policy satisfies

$$\frac{p_m^*}{q_m^* c'(q_m^*)} > 1$$

Next, note that from Lemma 2 we know that $\frac{p_0^*}{q_0^* c'(q_0^*)} = 1$, which then implies that

$$\frac{p_m^*}{q_m^* c'(q_m^*)} > \frac{p_0^*}{q_0^* c'(q_0^*)} \Leftrightarrow \frac{p_m^*}{p_0^*} > \frac{q_m^* c'(q_m^*)}{q_0^* c'(q_0^*)}.$$

Observe that for any convex increasing $c(q)$, if $q_m^* \geq q_0^*$ then $c'(q_m^*) \geq c'(q_0^*)$ and $\frac{q_m^* c'(q_m^*)}{q_0^* c'(q_0^*)} \geq \frac{q_m^*}{q_0^*}$. Therefore, if $q_m^* \geq q_0^*$ it also holds that

$$\frac{p_m^*}{p_0^*} > \frac{q_m^* c'(q_m^*)}{q_0^* c'(q_0^*)} \geq \frac{q_m^*}{q_0^*},$$

which implies $\frac{p_m^*}{q_m^*} > \frac{p_0^*}{q_0^*}$. \square

B. Proofs of Results in the Main Paper

Proof of Lemma 1 We show in turn that a threshold-type equilibrium exists and is unique. We then establish the properties of the purchasing threshold.

(i) Existence. If it exists, a threshold-type equilibrium requires that consumers with type $x_i \geq z$ ($x_i < z$) purchase a unit in the first period (delay purchase), for some $z \in \left[\frac{p}{q}, x_h \right]$. In turn, this implies that a mass

of $\beta\bar{G}(z)$ reviews are made available in the second period. Using Lemma 4, the equilibrium is specified by the following indifference equation

$$\begin{aligned} zq - p &= \delta \int_{\frac{p}{z}}^{\infty} (zs - p)f(s; q, z)ds \\ zq - p &= \delta \left(zq\bar{\Phi}\left(\frac{\frac{p}{z} - q}{\sigma_z}\right) + z\sigma_z\phi\left(\frac{\frac{p}{z} - q}{\sigma_z}\right) - p\bar{\Phi}\left(\frac{\frac{p}{z} - q}{\sigma_z}\right) \right) \end{aligned}$$

Let *rhs* (*lhs*) denote the right-hand side (left-hand side) of the last equation. Now, note that for $z \rightarrow x_h$, $\sigma_z \rightarrow 0$ and therefore *lhs* $\rightarrow x_h q - p$ and *rhs* $\rightarrow 0$; that is *lhs* $>$ *rhs*. By contrast for $z \rightarrow \frac{p}{q}$, we have *lhs* $\rightarrow 0$ and *rhs* $\rightarrow k$ for some $k > 0$; that is, *rhs* $>$ *lhs*. Continuity then implies that there exists some $z \in \left[\frac{p}{q}, x_h\right]$ that satisfies the indifference equation. Furthermore, from the last expression, we have for $Y := \frac{p-q}{\sigma_z}$,

$$-Y = \delta (\phi(Y) - Y\bar{\Phi}(Y)), \quad (9)$$

which is equivalent to $Y + \delta \int_Y^{\infty} \bar{\Phi}(s)ds = 0$.

(ii) Uniqueness. Rearranging expression (9), we have $\frac{Y}{Y\bar{\Phi}(Y) - \phi(Y)} = \delta$. Note that $\frac{Y}{Y\bar{\Phi}(Y) - \phi(Y)} = \frac{1}{1 - \bar{\Phi}(Y) - \frac{\phi(Y)}{Y}}$ is decreasing in Y (since $\frac{d}{dY} \left(1 - \bar{\Phi}(Y) - \frac{\phi(Y)}{Y}\right) = \phi(Y) \frac{1}{Y^2} > 0$), with $\lim_{Y \rightarrow -\infty} \left(\frac{Y}{Y\bar{\Phi}(Y) - \phi(Y)}\right) = 1$ and $\lim_{Y \rightarrow 0} \left(\frac{Y}{Y\bar{\Phi}(Y) - \phi(Y)}\right) = 0$. Therefore, there exists a unique value $Y < 0$ that solves (9).

We next establish that there exists a unique $z \in \left[\frac{p}{q}, x_h\right]$ that solves $Y = \frac{p-zq}{z\sigma_p \sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}}$. First note that for $z = \frac{p}{q}$ we have $\frac{p-zq}{z\sigma_p \sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}} = 0$ and that $\lim_{z \rightarrow x_h} \left(\frac{p-zq}{z\sigma_p \sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}}\right) = -\infty$ (since $\lim_{z \rightarrow x_h} \left(\sigma_p \sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}\right) = 0$). Therefore, by continuity there exists at least one $z \in \left[\frac{p}{q}, x_h\right]$ that solves the equation. Moreover, to see that such a solution is unique, note that

$$\frac{d}{dz} \left(\frac{p-zq}{z\sigma_p \sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}} \right) = \frac{\sigma_p \beta \gamma ((p-zq)zg(z) - 2p\bar{G}(z)(\bar{G}(z)\beta\gamma+1))}{2(\bar{G}(z)\beta\gamma+1)^2 z^2 \sigma_p^2 \left(\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}\right)^{\frac{3}{2}}} < 0,$$

where the last inequality holds because for $z \in \left[\frac{p}{q}, x_h\right]$ we have $p-zq < 0$.

(iii) Properties. (1) As δ increases, Y decreases and therefore z increases. (2) As β increases, the function $\frac{p-zq}{z\sigma_p \sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}}$ increases (since $\frac{d}{d\beta} \left(\frac{p-zq}{z\sigma_p \sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}}\right) = -\frac{p-zq}{2z\sigma_p \left(\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}\right) \sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}} \frac{\bar{G}(z)\gamma}{(\bar{G}(z)\beta\gamma+1)^2} > 0$) and therefore z increases. (3) As q (p) increases (decreases), the function $\frac{p-zq}{z\sigma_p \sqrt{\frac{\bar{G}(z)\beta\gamma}{\bar{G}(z)\beta\gamma+1}}}$ decreases and therefore z decreases.

Proof of Lemma 2 In the absence of SL, we have that the firm's total sales are $S(p, q) = 1 - G(\frac{p}{q})$. Moreover, $S_p = -\frac{1}{q}g(\frac{p}{q})$ and $S_q = \frac{p}{q^2}g(\frac{p}{q})$. Using Lemma 6, the optimal policy satisfies

$$c'(q) = -\frac{\frac{p}{q^2}g(\frac{p}{q})}{-\frac{1}{q}g(\frac{p}{q})} = \frac{p}{q}, \quad (10)$$

which for $c(q) = q^2$ becomes $p = 2q^2$. Inserting back into (7) and (8) in the proof of Lemma 6 we get the unique solution given in the proposition.

Proof of Proposition 1 We prove each of the two main points in the proposition in turn; we then prove an additional claim which does not appear in the main text, namely, that there exist intermediate levels of quality uncertainty where the firm's optimal policy satisfies both $q_m^* < q_0^*$ and $\frac{p_m^*}{q_m^*} > \frac{p_0^*}{q_0^*}$. Let $\hat{z} := z^{-1} = \frac{q}{p}$ so that $\sigma_z = \sigma_\epsilon \sqrt{\frac{(\hat{z}-a)\beta\gamma}{(\hat{z}-a)\beta\gamma+(b-a)}}$.

(i) Case small σ_ϵ . The firm's profit function when consumers are myopic can be written

$$\pi = \frac{1}{b-a}(p-q^2) \left(\frac{q}{p} - a + \frac{\sigma_z}{p} [\phi(0) - \phi(V) + V\bar{\Phi}(V)] \right),$$

where $V = \frac{bp-q}{\sigma_z}$. We will apply the implicit function theorem in the limit $\sigma_\epsilon \rightarrow 0$. To do so, we require the following derivatives. First, we have

$$\frac{\partial \pi}{\partial \sigma_\epsilon} = \frac{1}{b-a}(p-q^2) \left(\frac{1}{p} \frac{\partial \sigma_z}{\partial \sigma_\epsilon} [\phi(0) - \phi(V) + V\bar{\Phi}(V)] - \frac{1}{p} V \frac{\partial \sigma_z}{\partial \sigma_\epsilon} \bar{\Phi}(V) \right) = \frac{1}{b-a} \left(1 - \frac{q^2}{p} \right) \frac{\sigma_z}{\sigma_\epsilon} [\phi(0) - \phi(V)],$$

since $\frac{\partial \sigma_z}{\partial \sigma_\epsilon} = \frac{\sigma_z}{\sigma_\epsilon}$. Next, the cross-derivative with respect to p is

$$\begin{aligned} \frac{\partial^2 \pi}{\partial \sigma_\epsilon \partial p} &= \frac{1}{b-a} \left(\frac{q^2}{p^2} \frac{\sigma_z}{\sigma_\epsilon} [\phi(0) - \phi(V)] + \left(1 - \frac{q^2}{p} \right) \frac{1}{\sigma_\epsilon} \frac{\partial \sigma_z}{\partial p} [\phi(0) - \phi(V)] + \left(1 - \frac{q^2}{p} \right) \frac{\sigma_z}{\sigma_\epsilon} V \phi(V) \frac{\partial V}{\partial p} \right) \\ &= \frac{1}{b-a} \left(\frac{q^2}{p^2} \frac{\sigma_z}{\sigma_\epsilon} [\phi(0) - \phi(V)] - \left(1 - \frac{q^2}{p} \right) \frac{q}{p^2 \sigma_\epsilon} \frac{\partial \sigma_z}{\partial \hat{z}} [\phi(0) - \phi(V)] \right. \\ &\quad \left. + \left(1 - \frac{q^2}{p} \right) \frac{\sigma_z}{\sigma_\epsilon} V \phi(V) \left[\frac{b}{\sigma_z} + \frac{V}{\sigma_z} \frac{\partial \sigma_z}{\partial \hat{z}} \frac{q}{p^2} \right] \right). \end{aligned}$$

Noting that $\frac{\sigma_z}{\sigma_\epsilon} = \sqrt{\frac{(\hat{z}-a)\beta\gamma}{(\hat{z}-a)\beta\gamma+(b-a)}} := w_z$, $\frac{\partial \sigma_z}{\partial \hat{z}} = \frac{(b-a)\sigma_z}{2(\hat{z}-a)(\beta\gamma(\hat{z}-a)+b-a)} := h_z \sigma_z$, we may write

$$\begin{aligned} \frac{\partial^2 \pi}{\partial \sigma_\epsilon \partial p} &= \frac{1}{b-a} \left(\frac{q^2}{p^2} w_z [\phi(0) - \phi(V)] - \left(1 - \frac{q^2}{p} \right) \frac{q}{p^2} h_z w_z [\phi(0) - \phi(V)] \right. \\ &\quad \left. + \left(1 - \frac{q^2}{p} \right) w_z \left[\frac{bV\phi(V)}{\sigma_z} + V^2 \phi(V) h_z \frac{q}{p^2} \right] \right) \end{aligned}$$

Now consider $\lim_{\sigma_\epsilon \rightarrow 0} \frac{d^2 \pi}{d\sigma_\epsilon dp}$. Note that $\lim_{\sigma_\epsilon \rightarrow 0} \sigma_z = 0$, $\lim_{\sigma_\epsilon \rightarrow 0} V = \infty$. We can therefore calculate $\lim_{\sigma_\epsilon \rightarrow 0} \frac{V\phi(V)}{\sigma_z}$ as follows:

$$\begin{aligned} \lim_{\sigma_\epsilon \rightarrow 0} \frac{V\phi(V)}{\sigma_z} &= \lim_{\sigma_z \rightarrow 0} \frac{V\phi(V)}{\sigma_z} = \lim_{\sigma_z \rightarrow 0} \left[\frac{bp-q}{\sigma_z^2} \phi \left(\frac{bp-q}{\sigma_z} \right) \right] = \lim_{\sigma_z \rightarrow 0} \left[\frac{bp-q}{\sigma_z^2} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{bp-q}{\sigma_z} \right)^2} \right] \\ &= \frac{1}{(bp-q)\sqrt{2\pi}} \lim_{\sigma_z \rightarrow 0} \left[\left(\frac{bp-q}{\sigma_z} \right)^2 e^{-\frac{1}{2} \left(\frac{bp-q}{\sigma_z} \right)^2} \right] = \frac{1}{(bp-q)\sqrt{2\pi}} \lim_{\sigma_z \rightarrow 0} \frac{\left(\frac{bp-q}{\sigma_z} \right)^2}{e^{\frac{1}{2} \left(\frac{bp-q}{\sigma_z} \right)^2}} = \left(\frac{+\infty}{+\infty} \right) \\ &= \frac{1}{(bp-q)\sqrt{2\pi}} \lim_{\sigma_z \rightarrow 0} \frac{-2 \left(\frac{bp-q}{\sigma_z} \right) \frac{1}{\sigma_z^2}}{- \left(\frac{bp-q}{\sigma_z} \right) \frac{1}{\sigma_z^2} e^{\frac{1}{2} \left(\frac{bp-q}{\sigma_z} \right)^2}} = \frac{1}{(bp-q)\sqrt{2\pi}} \lim_{\sigma_z \rightarrow 0} \frac{2}{e^{\frac{1}{2} \left(\frac{bp-q}{\sigma_z} \right)^2}} = 0. \end{aligned}$$

Similarly, we have

$$\lim_{\sigma_\epsilon \rightarrow 0} V^2 \phi(V) = \lim_{\sigma_z \rightarrow 0} (bp-q) \frac{V\phi(V)}{\sigma_z} = 0,$$

so that

$$\lim_{\sigma_\epsilon \rightarrow 0} \frac{d^2 \pi}{d\sigma_\epsilon dp} = \frac{1}{b-a} \left(\frac{q^2}{p^2} - \left(1 - \frac{q^2}{p} \right) \frac{q}{p^2} h_z \right) w_z \phi(0).$$

Next, the cross-derivative with respect to q is

$$\begin{aligned} \frac{d^2\pi}{d\sigma_\epsilon dq} &= \frac{1}{b-a} \left(-\frac{2q}{p} \frac{\sigma_z}{\sigma_\epsilon} [\phi(0) - \phi(V)] + \left(1 - \frac{q^2}{p}\right) \frac{1}{\sigma_\epsilon} \frac{\partial \sigma_z}{\partial q} [\phi(0) - \phi(V)] + \left(1 - \frac{q^2}{p}\right) \frac{\sigma_z}{\sigma_\epsilon} V \phi(V) \frac{\partial V}{\partial q} \right) \\ &= \frac{1}{b-a} \left(-\frac{2q}{p} \frac{\sigma_z}{\sigma_\epsilon} [\phi(0) - \phi(V)] + \left(1 - \frac{q^2}{p}\right) \frac{1}{p\sigma_\epsilon} \frac{\partial \sigma_z}{\partial \hat{z}} [\phi(0) - \phi(V)] \right. \\ &\quad \left. + \left(1 - \frac{q^2}{p}\right) \frac{\sigma_z}{\sigma_\epsilon} V \phi(V) \left[-\frac{1}{\sigma_z} - \frac{V}{\sigma_z} \frac{\partial \sigma_z}{\partial \hat{z}} \frac{1}{p} \right] \right) \end{aligned}$$

Using the above definitions of w_z and h_z , we have

$$\begin{aligned} \frac{d^2\pi}{d\sigma_\epsilon dq} &= \frac{1}{b-a} \left(-\frac{2q}{p} w_z [\phi(0) - \phi(V)] + \left(1 - \frac{q^2}{p}\right) \frac{1}{p} h_z w_z [\phi(0) - \phi(V)] \right. \\ &\quad \left. + \left(1 - \frac{q^2}{p}\right) w_z \left[-\frac{V\phi(V)}{\sigma_z} - V^2 \phi(V) h_z \frac{1}{p} \right] \right) \end{aligned}$$

and then

$$\lim_{\sigma_\epsilon \rightarrow 0} \frac{d^2\pi}{d\sigma_\epsilon dq} = \frac{1}{b-a} \left(-\frac{2q}{p} + \left(1 - \frac{q^2}{p}\right) \frac{1}{p} h_z \right) w_z \phi(0).$$

Furthermore, it is straightforward to deduce the following:

$$\begin{aligned} \lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial \pi}{\partial p} &= \frac{1}{b-a} \left(\frac{q^3}{p^2} - a \right), \quad \lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial \pi}{\partial q} = \frac{1}{b-a} \left(1 - \frac{3q^2}{p} + 2aq \right), \quad \lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial^2 \pi}{\partial p^2} = \frac{1}{b-a} \left(-\frac{2q^3}{p^3} \right), \\ \lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial^2 \pi}{\partial q^2} &= \frac{1}{b-a} \left(-\frac{6q}{p} + 2a \right), \quad \lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial^2 \pi}{\partial q \partial p} = \frac{1}{b-a} \left(\frac{3q^2}{p^2} \right). \end{aligned}$$

Noting that $\lim_{\sigma_\epsilon \rightarrow 0} (p_m^*, q_m^*) = (p_0^*, q_0^*)$, we may now apply the implicit function theorem for small σ_ϵ . We have for p that

$$\begin{aligned} \lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial p}{\partial \sigma_\epsilon} &= \lim_{\sigma_\epsilon \rightarrow 0} \frac{-\frac{\partial^2 \pi}{\partial p \partial \sigma_\epsilon} \frac{\partial^2 \pi}{\partial q^2} + \frac{\partial^2 \pi}{\partial p \partial q} \frac{\partial^2 \pi}{\partial q \partial \sigma_\epsilon}}{\frac{\partial^2 \pi}{\partial p^2} \frac{\partial^2 \pi}{\partial q^2} - \left(\frac{\partial^2 \pi}{\partial p \partial q} \right)^2} \\ &= \frac{-\left(\frac{q^2}{p^2} - \left(1 - \frac{q^2}{p}\right) \frac{q}{p^2} h_z \right) w_z \phi(0) \left(-\frac{6q}{p} + 2a \right) + \left(\frac{3q^2}{p^2} \right) \left(-\frac{2q}{p} + \left(1 - \frac{q^2}{p}\right) \frac{1}{p} h_z \right) w_z \phi(0)}{\left(-\frac{2q^3}{p^3} \right) \left(-\frac{6q}{p} + 2a \right) - \left(\frac{3q^2}{p^2} \right)^2}. \end{aligned}$$

Evaluating the above at $(p_0^*, q_0^*) = \left(\frac{1}{8a^2}, \frac{1}{4a} \right)$, we have

$$\lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial p}{\partial \sigma_\epsilon} = \frac{-(4a^2 - 8a^3 h_z) w_z \phi(0) (-10a) + (12a^2) (-4a + 4a^2 h_z) w_z \phi(0)}{(-16a^3) (-10a) - (12a^2)^2} = \frac{(-8a^3 - 32a^4 h_z) w_z \phi(0)}{16a^4} < 0,$$

since $w_z, h_z > 0$. Similarly, we have for q that

$$\begin{aligned} \lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial q}{\partial \sigma_\epsilon} &= \lim_{\sigma_\epsilon \rightarrow 0} \frac{-\frac{\partial^2 \pi}{\partial q \partial \sigma_\epsilon} \frac{\partial^2 \pi}{\partial p^2} + \frac{\partial^2 \pi}{\partial p \partial q} \frac{\partial^2 \pi}{\partial p \partial \sigma_\epsilon}}{\frac{\partial^2 \pi}{\partial p^2} \frac{\partial^2 \pi}{\partial q^2} - \left(\frac{\partial^2 \pi}{\partial p \partial q} \right)^2} \\ &= \frac{-\left(-\frac{2q}{p} + \left(1 - \frac{q^2}{p}\right) \frac{1}{p} h_z \right) w_z \phi(0) \left(-\frac{2q^3}{p^3} \right) + \left(\frac{3q^2}{p^2} \right) \left(\frac{q^2}{p^2} - \left(1 - \frac{q^2}{p}\right) \frac{q}{p^2} h_z \right) w_z \phi(0)}{\left(-\frac{2q^3}{p^3} \right) \left(-\frac{6q}{p} + 2a \right) - \left(\frac{3q^2}{p^2} \right)^2} \end{aligned}$$

Evaluating the above at $(p_0^*, q_0^*) = \left(\frac{1}{8a^2}, \frac{1}{4a} \right)$, we have

$$\lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial q}{\partial \sigma_\epsilon} = \frac{-(-4a + 4a^2 h_z) w_z \phi(0) (-16a^3) + (12a^2) (4a^2 - 8a^3 h_z) w_z \phi(0)}{(-16a^3) (-10a) - (12a^2)^2}$$

$$= \frac{(-16a^4 - 32a^5 h_z) w_z \phi(0)}{16a^4} < 0$$

Finally we note that $Sign\left(\lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial}{\partial \sigma_\epsilon} \left(\frac{p}{q}\right)\right) = Sign\left(\lim_{\sigma_\epsilon \rightarrow 0} \left(q^* \frac{\partial p^*}{\partial \sigma_\epsilon} - p^* \frac{\partial q^*}{\partial \sigma_\epsilon}\right)\right)$. Evaluating the last expression we have

$$\begin{aligned} \lim_{\sigma_\epsilon \rightarrow 0} \left(q^* \frac{\partial p^*}{\partial \sigma_\epsilon} - p^* \frac{\partial q^*}{\partial \sigma_\epsilon}\right) &= \frac{1}{4a} \frac{(-8a^3 - 32a^4 h_z) w_z \phi(0)}{16a^4} - \frac{1}{8a^2} \frac{(-16a^4 - 32a^5 h_z) w_z \phi(0)}{16a^4} \\ &= \frac{(-2a^2 - 8a^3 h_z) w_z \phi(0)}{16a^4} - \frac{(-2a^2 - 4a^3 h_z) w_z \phi(0)}{16a^4} = \frac{-h_z w_z \phi(0)}{4a} < 0. \end{aligned}$$

(ii) Case large σ_ϵ . Consider the limit $\sigma_\epsilon \rightarrow \infty$. We have, for any $\frac{q}{p} \in [a, b]$

$$\begin{aligned} \lim_{\sigma_\epsilon \rightarrow \infty} \sigma_z &= \infty, \quad \lim_{\sigma_\epsilon \rightarrow \infty} V = \lim_{\sigma_\epsilon \rightarrow \infty} \frac{bp - q}{\sigma_z} = 0 \\ \lim_{\sigma_\epsilon \rightarrow \infty} S(p, q) &= \lim_{\sigma_\epsilon \rightarrow \infty} \frac{1}{b-a} \left(\frac{q}{p} - a + \frac{\sigma_z}{p} [\phi(0) - \phi(V) + V \bar{\Phi}(V)] \right) \\ &= \frac{1}{b-a} \left(\frac{q}{p} - a + \lim_{\sigma_\epsilon \rightarrow \infty} \frac{\sigma_z}{p} [\phi(0) - \phi(V)] + \left(b - \frac{q}{p}\right) \bar{\Phi}(0) \right) \\ &= \frac{1}{b-a} \left(\frac{1}{2} \frac{q}{p} + \frac{1}{2} b - a + \lim_{\sigma_\epsilon \rightarrow \infty} \frac{\sigma_z}{p} [\phi(0) - \phi(V)] \right) \end{aligned}$$

In the last expression, we have

$$\begin{aligned} \lim_{\sigma_\epsilon \rightarrow \infty} \sigma_z [\phi(0) - \phi(V)] &= \lim_{\sigma_z \rightarrow \infty} \frac{[\phi(0) - \phi(V)]}{\sigma_z^{-1}} = \lim_{\sigma_z \rightarrow \infty} \frac{V \phi(V) \frac{dV}{d\sigma_z}}{-\sigma_z^{-2}} \\ &= \lim_{\sigma_z \rightarrow \infty} \frac{-V \phi(V) \sigma_z^{-2} (bp - q)}{-\sigma_z^{-2}} = \lim_{\sigma_z \rightarrow \infty} \frac{(bp - q)^2 \phi(V)}{\sigma_z} = 0 \end{aligned}$$

Therefore, $\lim_{\sigma_\epsilon \rightarrow \infty} S(p, q) = \frac{1}{b-a} \left(\frac{1}{2} \frac{q}{p} + \frac{1}{2} b - a \right)$, and the firm's profit function becomes $\pi(p, q) = \frac{p-c(q)}{b-a} \left(\frac{q}{2p} + \frac{1}{2} b - a \right)$. We have

$$\begin{aligned} \pi_q &= \frac{1}{b-a} \left((p-c(q)) \left(\frac{1}{2p} \right) - c'(q) \left(\frac{q}{2p} + \frac{1}{2} b - a \right) \right) \\ \pi_p &= \frac{1}{b-a} \left((p-c(q)) \left(-\frac{q}{2p^2} \right) + \left(\frac{q}{2p} + \frac{1}{2} b - a \right) \right) = \frac{1}{b-a} \left(\frac{qc(q)}{2p^2} + \frac{1}{2} b - a \right). \end{aligned}$$

Observe that $\pi_p > 0$ for any $q \geq 0$, which implies that the optimal policy is such that $p = \frac{q}{a}$ (which is the highest possible price that generates positive sales at quality q). We may therefore restrict attention to the problem of maximizing profit subject to $q = ap$, i.e., maximizing $\pi(p) = \frac{p-c(ap)}{b-a} \left(\frac{ap}{2p} + \frac{1}{2} b - a \right)$. This is maximized at $c'(ap) = \frac{1}{a}$ and $\frac{p}{q} = \frac{1}{a}$. Moreover, using $c(q) = q^2$, we get $p^* = \frac{1}{2a^2}$, $q^* = \frac{1}{2a}$ and $\frac{p^*}{q^*} = \frac{1}{a}$.

(iii) We prove also that that for some intermediate values of σ_ϵ the optimal policy satisfies both $q_m^* < q_0^*$ and $\frac{p_m^*}{q_m^*} > \frac{p_0^*}{q_0^*}$. The result follows from the two points above and Lemma 7. First, we define the set

$$W = \{\sigma : \exists \epsilon > 0 \text{ s.t. } q_m^* < q_0^* \text{ for } \sigma_\epsilon \in [\sigma - \epsilon, \sigma), \text{ and } q_m^* \geq q_0^* \text{ for } \sigma_\epsilon \in [\sigma, \sigma + \epsilon]\}.$$

Note that by the two points above and continuity of the optimal policy in σ_ϵ , the set W is nonempty. Now, consider some $\sigma_\epsilon \in W$, and note that for this σ_ϵ we have by Lemma 7 that $\frac{p_m^*}{q_m^*} > \frac{p_0^*}{q_0^*}$. Next, consider σ'_ϵ , where $\sigma'_\epsilon = \sigma_\epsilon - k$ for $k > 0$. Since the optimal policy is continuous in σ_ϵ , it follows that for $\sigma_\epsilon \in W$ and sufficiently small k , we have that the optimal policy at σ'_ϵ satisfies $q_m^* < q_0^*$ and $\frac{p_m^*}{q_m^*} > \frac{p_0^*}{q_0^*}$ as required.

Proof of Corollary 1 Starting from the firm's optimal price and design policy in the absence of SL (i.e., $(p_0^*, q_0^*) = (\frac{1}{8a^2}, \frac{1}{4a})$; see Lemma 2), we assume that as β increases from zero, the firm adjusts either only the price or only the design of the product. We show in turn that (i) if the firm adjusts only the design, then the optimal design in the presence of SL satisfies $q_m^{**} < q_0^*$ (which implies $\frac{p_0^*}{q_m^{**}} > \frac{p_0^*}{q_0^*}$), and (ii) if the firm adjusts only price, then the optimal price in the presence of SL satisfies $p_m^{**} > p_0^*$ (which implies $\frac{p_m^{**}}{q_0^*} > \frac{p_0^*}{q_0^*}$).

Proof of (i). When the consumers are myopic, the firm's profit function is

$$\pi_m = \frac{1}{(b-a)} (p-c(q)) \left(b-a + \frac{\sigma_z}{p} (\phi(0) - \phi(V) - V\Phi(V)) \right)$$

where $V = \frac{bp-q}{\sigma_z} > 0$ and $\sigma_z = \sigma_p \sqrt{\frac{(\frac{q}{p}-a)\beta\gamma}{(\frac{q}{p}-a)\beta\gamma+(b-a)}}$. To show that $q_m^{**} < q_0^*$, it suffices to show that the profit function is concave in q for $p = p_0^*$, and that $\frac{d\pi_m}{dq}|_{q_0^*} < 0$.

We have (after some manipulation)

$$\frac{d\pi_m}{dq} = \frac{1}{(b-a)} \left(\frac{1}{p} \left((p-q^2) \frac{d\sigma_z}{dq} - 2q\sigma_z \right) (\phi(0) - \phi(V)) + \frac{1}{p} (p-3q^2+2bpq) \Phi(V) - 2q(b-a) \right)$$

and for $p = p_0^* = \frac{1}{8a^2}$,

$$\frac{d\pi_m}{dq} = \frac{1}{(b-a)} \left(\left((1-8a^2q^2) \frac{d\sigma_z}{dq} - 16a^2q\sigma_z \right) (\phi(0) - \phi(V)) + (1-24a^2q^2+2bq) \Phi(V) - 2q(b-a) \right).$$

Note that

$$\frac{d\sigma_z}{dq} = \frac{(b-a)p}{2(q-ap)((b-a)p+\beta\gamma(q-ap))} \sigma_z = \frac{4(b-a)a}{(8aq-1)(b-a+a\beta\gamma(8aq-1))} \sigma_z$$

so that

$$\frac{d\pi_m}{dq}|_{q=\frac{1}{4a}} = -\frac{1}{(b-a)} \left(\frac{2a(b-a+2a\beta\gamma)}{(b-a+a\beta\gamma)} \sigma_z (\phi(0) - \phi(V)) + \frac{b-a}{2a} (1-\Phi(V)) \right).$$

Therefore $\frac{d\pi_m}{dq}|_{q=\frac{1}{4a}} < 0$. Next, to show that π_m is concave in q let $S(q) = b-a + \frac{\sigma_z}{p} (\phi(0) - \phi(V) - V\Phi(V))$ and $\pi_m(q) = (p-c(q))S(q)$. We have

$$\frac{d^2\pi_m}{dq^2} = (p-c(q)) \frac{d^2S(q)}{dq^2} - c'(q) \frac{dS(q)}{dq} - c''(q)S(q).$$

Since $S(q) > 0$, $(p-c(q)) = (p-q^2) > 0$, $-c'(q) = -2q < 0$, and $-c''(q) = -2 < 0$, to prove concavity it is sufficient to show that $\frac{d^2S(q)}{dq^2} < 0$ and $\frac{dS(q)}{dq} > 0$. We have

$$\frac{dS(q)}{dq} = \frac{d\sigma_z}{dq} (\phi(0) - \phi(V)) + \Phi(V), \text{ and } \frac{d^2S(q)}{dq^2} = \frac{1}{p} \left(\frac{d^2\sigma_z}{dq^2} (\phi(0) - \phi(V)) - \frac{1}{\sigma_z} \left(\frac{d\sigma_z}{dq} V + 1 \right)^2 \phi(V) \right).$$

Since $\frac{d\sigma_z}{dq} > 0$ we have $\frac{dS(q)}{dq} > 0$. Moreover, since $\frac{d^2\sigma_z}{dq^2} = -\frac{p(b-a)((b-a)p+4\beta\gamma(q-ap))}{4(q-ap)^2((b-a)p+(q-ap)\beta\gamma)^2} \sigma_z < 0$, we have $\frac{d^2S(q)}{dq^2} < 0$.

Proof of (ii). To show that $p_m^{**} > p_0^*$, it suffices to show that the profit function π_m is concave in p for $q = q_0^*$, and that $\frac{d\pi_m}{dp}|_{p_0^*} > 0$.

We have (after some manipulation)

$$\frac{d\pi_m(p)}{dp} = \frac{1}{(b-a)} \left(b-a + \frac{1}{p} \left((p-q^2) \frac{d\sigma_z}{dp} + \frac{q^2}{p} \sigma_z \right) (\phi(0) - \phi(V)) - \left(b - \frac{q^3}{p^2} \right) \Phi(V) \right)$$

and for $q = q_0^* = \frac{1}{4a}$,

$$\frac{d\pi_m(p)}{dp}|_{q=\frac{1}{4a}} = \frac{1}{(b-a)} \left(b-a + \frac{1}{16a^2p} \left((16a^2p-1) \frac{d\sigma_z}{dp} + \frac{1}{p} \sigma_z \right) (\phi(0) - \phi(V)) - \left(b - \frac{1}{64a^3p^2} \right) \Phi(V) \right)$$

Note that

$$\frac{d\sigma_z}{dp} = -\frac{q(b-a)}{2(q-ap)((b-a)p + \beta\gamma(q-ap))} \sigma_z = -\frac{2a(b-a)}{(1-4a^2p)(4ap(b-a) + \beta\gamma(1-4a^2p))} \sigma_z$$

so that

$$\frac{d\pi_m(p)}{dp} \Big|_{p=\frac{1}{8a^2}} = \frac{1}{(b-a)} \left((b-a)(1-\Phi(V)) + \frac{4a^3\beta\gamma}{b-a+a\beta\gamma} \sigma_z (\phi(0) - \phi(V)) \right) > 0$$

It remains to show that π_m is concave in p . We have

$$\frac{d^2\pi_m(p)}{dp^2} = \frac{1}{(b-a)} \left(-2\frac{q^3}{p^3}\Phi(V) + \left(2\frac{q^2}{p^2}\frac{d\sigma_z}{dp} - 2\frac{q^2}{p^3}\sigma_z + \frac{1}{p}(p-q^2)\frac{d^2\sigma_z}{dp^2} \right) (\phi(0) - \phi(V)) - \frac{(p-q^2)}{p\sigma_z} \left(b - V\frac{d\sigma_z}{dp} \right)^2 \phi(V) \right),$$

which is negative provided $\left(2\frac{q^2}{p^2}\frac{d\sigma_z}{dp} - 2\frac{q^2}{p^3}\sigma_z + \frac{1}{p}(p-q^2)\frac{d^2\sigma_z}{dp^2} \right) < 0$. Noting that

$$\frac{d^2\sigma_z}{dp^2} = \frac{1}{4}q\sigma_z \frac{(b-a)}{(q-ap)^2} \frac{((b-a)(3q-4ap) - 4a\beta\gamma(q-ap))}{(((b-a)p + (q-ap)\beta\gamma))^2},$$

we get

$$\left(2\frac{q^2}{p^2}\frac{d\sigma_z}{dp} - 2\frac{q^2}{p^3}\sigma_z + \frac{1}{p}(p-q^2)\frac{d^2\sigma_z}{dp^2} \right) \Big|_{q=\frac{1}{4a}} = -\frac{1}{4}q \frac{\left(\frac{2}{a} \left(\frac{1-4a^2p}{4a} \right)^4 \beta^2\gamma^2 + 4 \frac{(1-4a^2p)(128a^4p^2-40a^2p+5)}{256a^4} (b-a)\beta\gamma p \right) + \frac{3}{64} \frac{128a^4p^2-48a^2p+5}{a^3} (b-a)^2 p^2}{p^3(q-ap)^2((b-a)p + (q-ap)\beta\gamma)^2} \sigma_z$$

Finally, since $128a^4p^2 - 48a^2p + 5 > 0$ for all p , it follows that $\frac{d^2\pi_m(p)}{dp^2} < 0$.

Proof of Proposition 2 Let $\pi_s(p, q)$ ($\pi_0(p, q)$) denote the firm's profit function in the presence (absence) of SL. In the presence of SL, we may write the firm's profit function as $\pi_s(p, q) = \frac{p-c(q)}{b-a} S(p, q)$, for $S(p, q) = b - a - \frac{\sigma_z}{p} (\phi(V) + V\Phi(V) - \phi(Y) - Y\Phi(Y))$. Note that from Lemma 1, we have $\lim_{\delta \rightarrow 1} z = x_h = a^{-1}$, $\lim_{\delta \rightarrow 1} \sigma_z = 0$, $\lim_{\delta \rightarrow 1} V = \infty$, and $\lim_{\delta \rightarrow 1} Y = -\infty$. Furthermore, it follows from the properties of the Normal distribution that $\lim_{\delta \rightarrow 1} \phi(V) = \lim_{\delta \rightarrow 1} \phi(Y) = \lim_{\delta \rightarrow 1} Y\Phi(Y) = 0$ (where the last can be shown using L'Hopital's rule). Therefore, we have $\lim_{\delta \rightarrow 1} S(p, q) = b - a - \frac{bp-q}{p} = \frac{q}{p} - a$, which then implies that $\lim_{\delta \rightarrow 1} \pi_s(p, q) = \pi_0(p, q)$. In turn, we have that $\lim_{\delta \rightarrow 1} (p_s^*, q_s^*) = (p_0^*, q_0^*)$, and the two claims of the proposition follow by continuity of the optimal policy in δ and by the comparison of (p_0^*, q_0^*) with (p_m^*, q_m^*) provided in Proposition 1.

Proof of Proposition 3 Using Lemma 5, we have

$$\pi_m(p, q) = \frac{p-c(q)}{b-a} \left(\frac{q}{p} - a + \frac{\sigma_z}{p} (\phi(0) - \phi(V) + V\bar{\Phi}(V)) \right),$$

for $V = \frac{bp-q}{\sigma_z}$. Invoking the envelope theorem, we have $\frac{d\pi^*}{d\beta} = \frac{\partial \pi^*}{\partial \beta}$ and

$$\begin{aligned} \frac{\partial \pi^*}{\partial \beta} &= \frac{p-c(q)}{b-a} \left(\frac{1}{p} \frac{\partial \sigma_z}{\partial \beta} (\phi(0) - \phi(V) + V\bar{\Phi}(V)) + \frac{\sigma_z}{p} \frac{\partial V}{\partial \beta} \bar{\Phi}(V) \right) \\ &= \frac{p-c(q)}{b-a} \left(\frac{1}{p} \frac{\partial \sigma_z}{\partial \beta} (\phi(0) - \phi(V) + V\bar{\Phi}(V)) + \frac{\sigma_z}{p} \frac{\partial V}{\partial \sigma_z} \frac{\partial \sigma_z}{\partial \beta} \bar{\Phi}(V) \right) \\ &= \frac{p-c(q)}{b-a} \left(\frac{1}{p} \frac{\partial \sigma_z}{\partial \beta} (\phi(0) - \phi(V) + V\bar{\Phi}(V)) - \frac{\sigma_z}{p} \frac{V}{\sigma_z} \frac{\partial \sigma_z}{\partial \beta} \bar{\Phi}(V) \right) = \frac{p-c(q)}{b-a} \left(\frac{1}{p} \frac{\partial \sigma_z}{\partial \beta} (\phi(0) - \phi(V)) \right) > 0, \end{aligned}$$

where the last inequality holds because $\frac{\partial \sigma_z}{\partial \beta} > 0$. Using the same argument, it is straightforward to deduce that the result holds also for parameters γ and σ_ϵ .

Proof of Proposition 4 Using Lemma 5, we write the firm's profit function as

$$\pi(p, q) = \frac{p - c(q)}{(b - a)} \left(\frac{1}{z} - a + \frac{\sigma_z}{p} (\phi(Y) - \phi(V) + V\bar{\Phi}(V) - Y\bar{\Phi}(Y)) \right),$$

for $V = \frac{bp - q}{\sigma_z}$ and $Y = \frac{pz^{-1} - q}{\sigma_z}$. Let π^* denote the firm's optimal profit. Invoking the envelope theorem, we have $\frac{d\pi^*}{d\beta} = \frac{\partial\pi^*}{\partial\beta}$, and

$$\begin{aligned} \frac{\partial\pi}{\partial\beta} &= -\frac{1}{z^2} \frac{\partial z}{\partial\beta} + \frac{1}{p} \frac{\partial\sigma_z}{\partial\beta} (\phi(Y) - \phi(V) + V\bar{\Phi}(V) - Y\bar{\Phi}(Y)) + \frac{\sigma_z}{p} \left(\frac{\partial V}{\partial\beta} \bar{\Phi}(V) - \frac{\partial Y}{\partial\beta} \bar{\Phi}(Y) \right) \\ &= -\frac{1}{z^2} \frac{\partial z}{\partial\beta} + \frac{1}{p} \frac{\partial\sigma_z}{\partial\beta} (\phi(Y) - \phi(V)) + \frac{\sigma_z}{p} \left(\frac{p}{z^2 \sigma_z} \frac{\partial z}{\partial\beta} \bar{\Phi}(Y) \right) \\ &= \frac{1}{z^2} \frac{\partial z}{\partial\beta} (\bar{\Phi}(Y) - 1) + \frac{1}{p} \frac{\partial\sigma_z}{\partial\beta} (\phi(Y) - \phi(V)), \end{aligned}$$

where we have used $\frac{\partial V}{\partial\beta} = -\frac{V}{\sigma_z} \frac{\partial\sigma_z}{\partial\beta}$ and $\frac{\partial Y}{\partial\beta} = -\frac{Y}{\sigma_z} \frac{\partial\sigma_z}{\partial\beta} - \frac{p}{z^2 \sigma_z} \frac{\partial z}{\partial\beta}$. Now, note that from Lemma 1 we have

$$\frac{\partial z}{\partial\beta} = -\frac{Y \frac{\partial\sigma_z}{\partial\beta}}{\frac{p}{z^2} + Y \frac{\partial\sigma_z}{\partial z}} > 0$$

and that

$$\frac{\partial\sigma_z}{\partial\beta} = -\frac{d\sigma_z}{dz} \frac{Y \frac{\partial\sigma_z}{\partial\beta}}{\left(\frac{p}{z^2} + Y \frac{d\sigma_z}{dz}\right)} + \frac{\partial\sigma_z}{\partial\beta} = \frac{-Y \frac{\partial\sigma_z}{\partial\beta} \frac{d\sigma_z}{dz} + \frac{\partial\sigma_z}{\partial\beta} \left(\frac{p}{z^2} + Y \frac{d\sigma_z}{dz}\right)}{\left(\frac{p}{z^2} + Y \frac{d\sigma_z}{dz}\right)} = \frac{\frac{\partial\sigma_z}{\partial\beta} \frac{p}{z^2}}{\left(\frac{p}{z^2} + Y \frac{d\sigma_z}{dz}\right)} > 0.$$

Therefore, it follows that if $\phi(Y) < \phi(V)$, then $\frac{\partial\pi}{\partial\beta} < 0$. Since $Y < 0$ and $V > 0$, this occurs provided $q^* - \frac{p^*}{z} > bp^* - q^*$ or, equivalently, provided $z > \frac{p^*}{2q^* - bp^*}$. Now, recall from Lemma 1 that $z \in [\frac{p^*}{q^*}, x_h]$, with $\frac{dz}{d\delta} > 0$ and $\lim_{\delta \rightarrow 1} z = x_h$. Therefore, for sufficiently high δ , we have $z > \frac{p^*}{2q^* - bp^*}$ provided that $x_h = \frac{1}{a} > \frac{p^*}{2q^* - bp^*}$, or equivalently, provided $\frac{q^*}{p^*} > \frac{(a+b)}{2}$. Now recall that $\lim_{\delta \rightarrow 1} (p^*, q^*) = (p_0^*, q_0^*)$ (see proof of Proposition 2). Therefore, if $\frac{q_0^*}{p_0^*} > \frac{(a+b)}{2}$, then for sufficiently large δ we have $\frac{q^*}{p^*} > \frac{(a+b)}{2}$. From Lemma 2, we have $p_0^* = \frac{1}{8a^2}$ and $q_0^* = \frac{1}{4a}$, so that $\frac{q_0^*}{p_0^*} = 2a$. Thus, for the claim of the proposition to hold, we require $\frac{q_0^*}{p_0^*} = 2a > \frac{(a+b)}{2}$, which is equivalent to the condition $3a > b$.

In addition to the proof of the result in the main text, we now also prove that the result of the proposition continues to hold unchanged when the firm adjusts either only the price or only the design of the product in the presence of SL (i.e., holding the other decision variable fixed). We first prove the case where the firm adjusts only the price; in this case, let p_s^{**} denote the optimal price in the presence of SL. As in the main proof above, we have that if $\phi(Y) < \phi(V)$, then $\frac{\partial\pi_s(p)}{\partial\beta} < 0$. Now since $Y < 0$ and $V > 0$ this occurs when $q - \frac{p_s^{**}}{z} > bp_s^{**} - q$ or equivalently when $z > \frac{p_s^{**}}{2q - bp_s^{**}}$. Now, $z \in \left[\frac{p_s^{**}}{q}, \frac{1}{a}\right]$ and $\lim_{\delta \rightarrow 1} z = \frac{1}{a}$. Therefore, for sufficiently high δ ; if $\frac{p_s^{**}}{2q - bp_s^{**}} < \frac{1}{a}$ then $z > \frac{p_s^{**}}{2q - bp_s^{**}}$. The last equation holds when $p_s^{**} < \frac{2q}{(a+b)}$. Moreover, $\lim_{\delta \rightarrow 1} p_s^{**} = p_0^* = \frac{1}{8a^2}$. Therefore, if $\frac{1}{8a^2} < \frac{2q}{(a+b)}$ then for sufficiently high δ we also have $p_s^{**} < \frac{2q}{(a+b)}$ as required. Finally, note that for $q = q_0^* = \frac{1}{4a}$ and $3a > b$, the condition $p_0^* < \frac{2q}{(a+b)}$ holds. When the firm adjusts only the design, let q_s^{**} denote the optimal design in the presence of SL. Here, we again have that if $\phi(Y) < \phi(V)$, then $\frac{\partial\pi_s(q)}{\partial\beta} < 0$. Now since $Y < 0$ and $V > 0$ this occurs when $q_s^{**} - \frac{p}{z} > bp - q_s^{**}$ or equivalently when $z > \frac{p}{2q_s^{**} - bp}$. Now, $z \in \left[\frac{p}{q_s^{**}}, \frac{1}{a}\right]$ and $\lim_{\delta \rightarrow 1} z = \frac{1}{a}$. Therefore, for sufficiently high δ ; if $\frac{p}{2q_s^{**} - bp} < \frac{1}{a}$ then $z > \frac{p}{2q_s^{**} - bp}$. The last equation holds when $\frac{(a+b)}{2} p < q_s^{**}$. Moreover, $\lim_{\delta \rightarrow 1} q_s^{**} = q_0^* = \frac{1}{4a}$. Therefore, if $\frac{1}{4a} > \frac{(a+b)}{2} p$ then for sufficiently high δ we also have $q_s^{**} > \frac{(a+b)}{2} p$ as required. Finally, note that for $p = p_0^* = \frac{1}{8a^2}$ and $3a > b$, the condition $\frac{1}{4a} > \frac{(a+b)}{2} p$ holds.

Proof of Lemma 3 In the absence of SL, the firm chooses (p_0^*, q_0^*) with $\frac{p_0^*}{q_0^*} = \frac{1}{2a}$ (see Lemma 2). Thus, in the absence of SL, every consumer with $x_i \leq \frac{1}{2a}$ derives zero utility. By contrast, in the presence of SL every such consumer either (a) purchases in the first period (which implies a positive expected utility) or (b) waits until the second period, in which case she derives expected utility equal to $\delta \int_{\frac{p}{x_i}}^{\infty} (x_i s - p) f(s; q, z) ds > 0$, where z is the first-period purchase threshold at the firm's chosen policy. Therefore, any customer with $x_i \leq \frac{1}{2a}$ is better off in the presence of SL, while customers with $x_i > \frac{1}{2a}$ may or may not be better off, depending on the firm's chosen policy. The statement in the proposition then follows.

Proof of Proposition 5 The consumers' expected surplus is given by

$$S(p, q) = \int_z^{x_h} (xq - p)g(x)dx + \delta \int_{\frac{p}{z}}^{\infty} \int_{\max\{\frac{p}{s}, x_i\}}^z (xs - p)g(x)f(s; q, z)dx ds,$$

where the surplus of first-period buyers is given by

$$S_1 := \int_z^{x_h} (xq - p)g(x)dx = \frac{1}{b-a} \left[q \ln\left(\frac{x_h}{z}\right) + p \left(\frac{1}{x_h} - \frac{1}{z}\right) \right].$$

Note that in the extreme case of $\delta = 0$, we have $z = \frac{p}{q}$ and the above expression becomes $S(p, q) = \int_{\frac{p}{q}}^{x_h} (xq - p)g(x)dx$. We prove each point in the proposition in turn.

Consider first the case of small σ_ϵ . We evaluate $\lim_{\sigma_\epsilon \rightarrow 0} \frac{dS(p^*, q^*)}{d\sigma_\epsilon}$, where (p^*, q^*) is the firm's optimal policy. To do so, we use the proof of Proposition 1 (see definitions of w_z , h_z and $\frac{dp}{d\sigma_\epsilon}$, $\frac{dq}{d\sigma_\epsilon}$). In particular, we have

$$\lim_{\sigma_\epsilon \rightarrow 0} \frac{dS}{d\sigma_\epsilon} = \lim_{\sigma_\epsilon \rightarrow 0} \left(\frac{\partial S}{\partial \sigma_\epsilon} + \frac{\partial S}{\partial q} \frac{\partial q}{\partial \sigma_\epsilon} + \frac{\partial S}{\partial p} \frac{\partial p}{\partial \sigma_\epsilon} \right),$$

where $\frac{\partial S}{\partial \sigma_\epsilon} = 0$ and

$$\begin{aligned} \lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial S}{\partial q} &= \frac{\ln\left(\frac{1}{a}\right) - \ln\left(\frac{p_0^*}{q_0^*}\right)}{b-a} = \frac{\ln(2)}{b-a}, & \lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial S}{\partial p} &= -\frac{\frac{q_0^*}{p_0^*} - a}{b-a} = -\frac{a}{b-a}, \\ \lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial q}{\partial \sigma_\epsilon} &= (-1 - 2ah_z)w_z\phi(0), & \lim_{\sigma_\epsilon \rightarrow 0} \frac{\partial p}{\partial \sigma_\epsilon} &= (-1 - 2ah_z)w_z\phi(0). \end{aligned}$$

Therefore,

$$\lim_{\sigma_\epsilon \rightarrow 0} \frac{dS}{d\sigma_\epsilon} = \frac{w_z\phi(0)}{b-a} \left(\ln(2) [-1 - 2ah_z] - a \left[-\frac{1}{2a} - 2h_z \right] \right) = \frac{w_z\phi(0)}{b-a} \left(\frac{1}{2} - \ln(2) + 2ah_z(1 - \ln(2)) \right),$$

which, since $w_z, h_z > 0$ and $h_z = \frac{(b-a)}{2\left(\frac{q_0^*}{p_0^*} - a\right)\left(\beta\gamma\left(\frac{q_0^*}{p_0^*} - a\right) + b - a\right)}$, is positive if and only if

$$\begin{aligned} 1 - 2\ln(2) + 4ah_z(1 - \ln(2)) > 0 &\Leftrightarrow h_z > \frac{2\ln(2) - 1}{4a(1 - \ln(2))} \Leftrightarrow \frac{(b-a)}{2a(\beta\gamma a + b - a)} > \frac{2\ln(2) - 1}{4a(1 - \ln(2))} \Leftrightarrow \\ \beta &< \left(\frac{3 - 4\ln(2)}{2\ln(2) - 1} \right) \left(\frac{b-a}{a\gamma} \right) = 0.589 \left(\frac{b-a}{a\gamma} \right). \end{aligned}$$

Consider next the case large σ_ϵ . Notice that since $\lim_{\sigma_\epsilon \rightarrow \infty} \frac{p_0^*}{q_0^*} = \frac{1}{a}$ this case is trivial: the consumers' expected surplus is zero in the presence of SL, while it is positive in its absence.

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